Review Topic 1: Factoring and Solving Quadratics

7 Factoring Rules

Rule #1 - GCF

1) $3a^2bc^5 - 9a^3bc^6 + 12a^2b^3c^6$

$3a^2bc^5(1 - 3ac + 4b^2c)$

Rule #2 – Difference of Perfect Squares

2) $9x^2 - 49y^2$

$(3x+7y)(3x-7y)$

3) $100x^2 - 100$

$100(x^2-1)$

$100(x+1)(x-1)$

Rule #3 – Trinomial with 1 as a Leading Coefficient

4) $a^2 + a - 20$

$(a+5)(a-4)$

5) $3x^2 - 18x + 24$

$3(x^2 - 6x + 8)$

$3(x-4)(x-2)$

Rule #4 – Sum/Difference of Perfect Cubes

6) $x^3 - 125y^3$

$(x-5y)(x^2+5xy+25y^2)$

7) $64a^3 + 8y^3$

$8(8a^3+y^3)$

$8(2a+y)(4a^2-2ay+y^2)$

Rule #5 – Perfect Square Trinomials

8) $9x^2 + 30x + 25$

$(3x+5)^2$

9) $36y^2 - 84y + 49$

$(6y-7)^2$

Rule #6 – Factoring by Grouping

10) $\underbrace{a^2x - b^2x}\underbrace{+ a^2y - b^2y}$

$x(a^2-b^2)+y(a^2-b^2)$

$(a^2-b^2)(x+y)$

$(a+b)(a-b)(x+y)$

11) $4x^3 + 4x^2 - 6x + 6$

$2(\underbrace{2x^3+2x^2}\underbrace{-3x+3})$

* Cannot be factored further

Rule #7 – Trinomial with Leading Coefficient >1

12) $3y^2 + 5y + 2$

$(3y+2)(y+1)$

13) $6x^2 - 5x - 6$

$(3x+2)(2x-3)$

What if it can't be factored?

14) $x^2 + 25y^2$

prime

How do I know which way to solve?

Try to solve by factoring first, if you can't solve by factoring use the quadratic formula or solve by completing the square. Check your answer with your graphing calc! Graph the quadratic and make sure your answers are the x-intercepts!

Solving Quadratics Mixed Practice

26) $x^2+4x=3$

$x^2+4x-3=0$

$x^2+4x+4=3+4$

$\sqrt{(x+2)^2}=\pm\sqrt{7}$

$x+2=\pm\sqrt{7}$

$x=-2\pm\sqrt{7}$

27) $2x^2+5x=3$

$2x^2+5x-3=0$

$(2x-1)(x+3)=0$

$2x-1=0 \quad x+3=0$

$x=\frac{1}{2} \quad x=-3 \quad \{-3,\frac{1}{2}\}$

28) $x^2=5x$

$x^2-5x=0$

$x(x-5)=0$

$x=0 \quad x-5=0$

$x=5 \quad \{0,5\}$

29) $x^2+3x-40=0$

$(x+8)(x-5)=0$

$x+8=0 \quad x-5=0 \quad \{-8,5\}$

$x=-8 \quad x=5$

30) $3x^2-5x+2=0$

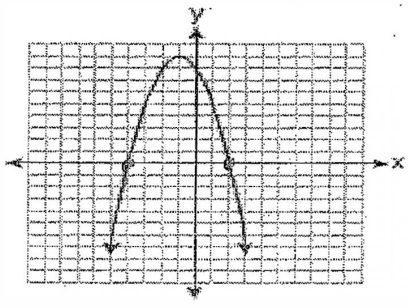
$(3x+1)(x-2)=0$

$3x+1=0 \quad x-2=0$

$3x=-1 \quad x=2$

$x=-\frac{1}{3} \quad \{-\frac{1}{3},2\}$

31)



$\{-4,2\}$

Describe the Nature of the Roots of a Quadratic

A) 2 Real Rational Roots
B) 1 Real Double Root
C) 2 Real Irrational Roots
D) 2 Imaginary Roots

What are the nature of the roots of the following quadratics with roots?

32) $\left\{\pm\frac{1}{3}\right\}$

2 Real Rational

33) $\left\{\pm\frac{1}{3}i\right\}$

2 Imaginary

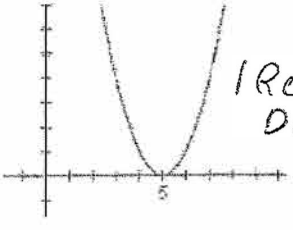
34) $\{0,-5\}$

2 Real Rational

35) $\{2\pm\sqrt{5}\}$

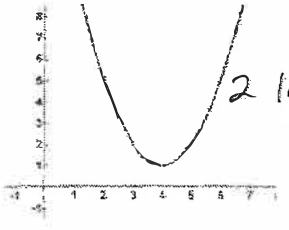
2 Real Irrational

36)



1 Real Double

37)



2 Imaginary

SOL Review Topic 2: Rational Expressions and Equations

Simplifying Rational Expressions

Factor and Reduce!

1) $\frac{2x^2+3x-2}{2x^2-8}$ $2(x^2-4)$ $= \frac{(2x-1)\cancel{(x+2)}}{2\cancel{(x+2)}(x-2)} = \frac{2x-1}{2(x-2)} = \frac{2x-1}{2x-4}$

Multiplying and Dividing Rational Expressions

2) $\frac{x^2-1}{1+x} \div \frac{2x^2-5x+3}{3-2x}$ $= -1$

$\frac{(x+1)(x-1)}{(x+1)} \cdot \frac{-2x+3}{(2x-3)(x-1)} = \frac{\cancel{(x+1)}\cancel{(x-1)} \cdot -1\cancel{(2x-3)}}{\cancel{(x+1)}\cancel{(2x-3)}\cancel{(x-1)}}$

3) $\frac{\frac{6x+12}{5x}}{\frac{x+2}{10x}}$ $\frac{6\cancel{(x+2)}}{\cancel{5x}} \cdot \frac{10\cancel{x}^{2}}{\cancel{x+2}} = 12$

Adding and Subtracting Rational Expressions

YOU NEED A COMMON DENOMINATOR WHENEVER YOU ARE ADDING OR SUBTRACTING FRACTIONS!

Do not cancel on top and bottom! Get a common denominator & then add or subtract the numerators.

4) $\frac{8}{(x+2)(x-2)} - \frac{4}{(x+2)(x-2)}$ LCD: $(x+2)(x-2)$

$\frac{8x+16-4x+8}{(x+2)(x-2)} = \frac{4x+24}{(x+2)(x-2)}$

5) $\frac{c+d}{2cd^2} + \frac{d-c}{6c^2d}$ LCD: $6c^2d^2$

$\frac{(3c)(c+d)}{(3c)(2cd^2)} + \frac{(-c+d)(d)}{(6c^2d)(d)} = \frac{3c^2+3cd-cd+d^2}{6c^2d^2}$

$\frac{3c^2+2cd+d^2}{6c^2d^2}$

Solving Rational Equations

Multiply both sides of the equation by the LCD then cancel the fractions and simplify. f

6) $\frac{x+1}{3(x-2)} = \frac{5x}{6} + \frac{1}{x-2}$ $6(x-2)$

$2(x+1) = 5x(x-2)+6$

$2x+2 = 5x^2-10x+6$

$0 = 5x^2-12x+4$

$(5x-2)(x-2)=0$

$5x-2=0$

$x=\frac{2}{5}$

$x-2=0$

$\cancel{x=2}$ Excluded

$\left\{\frac{2}{5}\right\}$

7) $\frac{4}{u+2} - \frac{2}{u-2} = \frac{u}{u^2-4}$ $(u+2)(u-2)$

$4(u-2)-2(u+2)=u$

$4u-8-2u-4=u$

$2u-12=u$

$u=12$

Mixed Practice Simplifying and Adding and Subtracting Rational Expressions

8) $\frac{x+7}{x-2} - \frac{4}{x^2+x-6} = 1$ $(x-2)(x+3)$

$(x+7)(x+3)-4=(x-2)(x+3)$

$\cancel{x^2}+10x+21-4=\cancel{x^2}+x-6$

$9x=-23$

$x=-\frac{23}{9}$

9) $\frac{x^2-7x+12}{x^2-x-6} \quad \frac{x^2+7x+10}{x^2+x-20}$

$\frac{\cancel{(x-3)}\cancel{(x-4)} \cdot \cancel{(x+5)}\cancel{(x+2)}}{\cancel{(x-3)}\cancel{(x+2)} \cdot \cancel{(x+5)}\cancel{(x-4)}} = 1$

10) $\frac{2-5x}{x-9} + \frac{4x-5}{9-x}$

$\frac{-5x+2}{x-9} + \frac{4x-5}{-1(x-9)}$

$\frac{-5x+2-(4x-5)}{x-9} = \frac{-9x+7}{x-9}$

11) $\frac{x^2-9}{4} \div \frac{3-x}{8}$

$\frac{(x+3)\cancel{(x-3)}}{\cancel{4}} \cdot \frac{\cancel{8}^{2}}{-1\cancel{(x-3)}} = -2(x+3)$

SOL Review Topic 3: Simplifying Radicals and Complex Numbers

Simplifying Radicals

1) $\sqrt{200a^2b^2c^{11}}$

$\sqrt{100\cdot 2\cdot a^2\cdot b^2\cdot c^{10}\cdot c}$

$10|a|b|c^5\sqrt{2c}$

2) $\sqrt[3]{54} = \sqrt[3]{27\cdot 2}$

$3\sqrt[3]{2}$

3) $\sqrt[4]{16a^5b^{11}}$

$\sqrt[4]{16\cdot a^4\cdot a\cdot b^8\cdot b^3}$

$2|a|b^2\sqrt[4]{ab^3}$

4) $\sqrt[3]{-8a^4b^9c^{11}}$

$\sqrt[3]{-1\cdot 8\cdot a^3\cdot a\cdot b^9\cdot c^9\cdot c^2}$

$-2ab^3c^3\sqrt[3]{ac^2}$

Operations with Radicals

5) $\sqrt{72x^3y}\cdot\sqrt{50xy^3}$

$\sqrt{36\cdot 2\cdot 25\cdot 2\cdot x^4\cdot y^4}$

$6\cdot 5\cdot 2\cdot x^2\cdot y^2$

$60x^2\cdot y^2$

6) $(2-\sqrt{3})(2+\sqrt{3})$

$4-\sqrt{9}$

$4-3$

1

7) $\sqrt[3]{24}-\sqrt[3]{81}+\sqrt[3]{3}$

$\sqrt[3]{8\cdot 3}-\sqrt[3]{27\cdot 3}+\sqrt[3]{3}$

$2\sqrt[3]{3}-3\sqrt[3]{3}+\sqrt[3]{3}$

0

8) $\frac{1}{\sqrt[4]{2}}\cdot\frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}} = \frac{\sqrt[4]{2^3}}{2}$

9) $\sqrt{12x^5}-\sqrt{8x^5}$

$\sqrt{4\cdot 3\cdot x^4\cdot x}-\sqrt{4\cdot 2\cdot x^4\cdot x}$

$2x^2\sqrt{3x}-2x\sqrt{2x}$

10) $\frac{2}{(3-\sqrt{2})}\frac{(3+\sqrt{2})}{(3+\sqrt{2})}$

$\frac{6+2\sqrt{2}}{9-2} = \frac{6+2\sqrt{2}}{7}$

Rational Exponents

Express the following in exponential form:

11) $\sqrt[3]{(3x)^2}$

$(3x)^{\frac{2}{3}}$

★ $\sqrt[3]{3x^2}$

$=3^{\frac{1}{3}}x^{\frac{2}{3}}$

12) $\sqrt{7} = 7^{\frac{1}{2}}$

13) $\sqrt[4]{4a^5}$

$4^{\frac{1}{4}}a^{\frac{5}{4}}$

14) $\sqrt[4]{(4a)^5}$

$(4a)^{\frac{5}{4}}$

Express the following in radical form:

15) $r^{-\frac{1}{3}} = \frac{1}{r^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{r}} = \frac{1}{\sqrt[3]{r}}\cdot\frac{\sqrt[3]{r^2}}{\sqrt[3]{r^2}} = \frac{\sqrt[3]{r^2}}{r}$

17) $a^{\frac{2}{3}}b^{\frac{1}{2}} = a^{\frac{4}{6}}b^{\frac{3}{6}} = \sqrt[6]{a^4b^3}$

Powers of *i*

$i^{\text{any multiple of 4}} = 1$

18) $i = \sqrt{-1} = i$

$i^2 = (\sqrt{-1})^2 = -1$

$i^3 = i^2\cdot i = -i$

$i^4 = 1$

$i^5 = i$

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

19) $i^{33} = i$

$i^{32} = 1$ therefore

20) $i^{102} = -1$

$i^{100} = 1$ therefore

$i^{101} = i$

Imaginary Numbers

21) $\sqrt{-50}$

$\sqrt{-25\cdot 2}$

$5i\sqrt{2}$

22) $\sqrt{-3}\cdot\sqrt{-3}$

$\sqrt{-1\cdot 3}\cdot\sqrt{-1\cdot 3}$

$i\sqrt{3}\cdot i\sqrt{3}$

$i^2\sqrt{9}$

$-1\cdot 3 = -3$

23) $\frac{2}{(2-3i)}\frac{(2+3i)}{(2+3i)}$

$\frac{4+6i}{4-9i^2} = \frac{4+6i}{4+9}$

$\frac{4+6i}{13}$

Operations with Complex Numbers

24) $(2+i)(4-3i)$

$8-6i+4i-3i^2$

$8-2i+3$

$11-2i$

25) $(1-5i\sqrt{3})-(4+i\sqrt{3})+i$

$-3-5i\sqrt{3}+i$

Solving Radical and Rational Exponent Equations

26) $(5a-5)^{\frac{1}{3}}+1=3$

$\left((5a-5)^{\frac{1}{3}}\right)^3=(2)^3$

$5a-5=8$

$5a=13$

$a=\frac{13}{5}$

27) $\sqrt{x-1}+2=-1$

$\sqrt{x-1}=-3$

No Solution

28) $\sqrt[5]{\frac{1}{5}x-7}-1=-2$

$\left(\sqrt[5]{\frac{1}{5}x-7}\right)^5=(-1)^5$

$\frac{1}{5}x-7=-1$

$\frac{1}{5}x=6$

$x=30$

Mixed Practice with Radicals and Complex Numbers

SIMPLIFY OR SOLVE:

$i \quad -1 \quad -i \quad 1$

29) $-2\sqrt[4]{4x-12}+1=-3$

$-2\sqrt[4]{4x-12}=-4$

$\left(\sqrt[4]{4x-12}\right)^4=(2)^4$

$4x-12=16$

$4x=4 \quad x=1$

30) $x^2+49=0$

$x^2=-49$

$\sqrt{x^2}=\pm\sqrt{-49}$

$x=\pm 7i$

31) i^{93}

$i^{92}=1 \quad i^{93}=i$

32) $\sqrt{-7}\cdot\sqrt{-7}$

$i\sqrt{7}\cdot i\sqrt{7}$

$i^2\sqrt{49}$

-49

33) $4\sqrt[4]{16p^4q^9}$

$4\cdot 2|p|q^2\sqrt[4]{q}$

$8|p|q^2\sqrt[4]{q}$

34) $\sqrt{24p^3q^{12}r^{24}}$

$\sqrt{4\cdot 6\cdot p^2\cdot p\cdot q^{12}\cdot r^{24}}$

$2pq^6r^{12}\sqrt{6p}$

35) $(2+\sqrt{5})(3-2\sqrt{5})$

$6-4\sqrt{5}+3\sqrt{5}-2\sqrt{25}$

$6-1\sqrt{5}-10$

$-4-\sqrt{5}$

36) $3\sqrt{75}-12\sqrt{3}-\sqrt{18}+5\sqrt{8}$

$3\sqrt{25\cdot 3}-12\sqrt{3}-\sqrt{9\cdot 2}+5\sqrt{4\cdot 2}$

$15\sqrt{3}-12\sqrt{3}-3\sqrt{2}+10\sqrt{2}$

$3\sqrt{3}+7\sqrt{2}$

37) $3(x-2)^{\frac{3}{4}}+1=25$

$3(x-2)^{\frac{3}{4}}=24$

$\left((x-2)^{\frac{3}{4}}\right)^{\frac{4}{3}}=(8)^{\frac{4}{3}}$

$x-2=16$

$x=18$

SOL Review Topic 4: Solving Equations and Inequalities

Absolute Value Equations, Rational Equations, Radical Equations, Quadratics, Systems, Inequalities

Absolute Value Equations

1) $3|2x-7|+2=17$

$3|2x-7| = 15$

$|2x-7| = 5$

$2x-7=5$ or $2x-7=-5$

$x=6$ or $x=1$

2) $|9+4x|=5x+18$

$9+4x=5x+18$ or $9+4x=-(5x+18)$

$-9=x$ or $9+4x=-5x-18$

$9x=-27$

$x=-3$ ✓

$\cancel{x=-9}$

$x=-9$ is extraneous

*Must check for extraneou solutions!

Absolute Value Inequalities

Great OR Less THAN

3) $\frac{1}{2}|x-8|-3\le 16$

$\frac{1}{2}|x-8| \le 19$

$|x-8| \le 38$

$x-8\le 38$ and $x-8\ge -38$

$x\le 46$ and $x\ge -30$

4) $|x-3|-2\le 1$

$|x-3|\le 3$

$x-3\le 3$ and $x-3\ge -3$

$x\le 6$ and $x\ge 0$

Radical and Rational Exponent Equations

5) $2=\frac{1}{3}\sqrt{5x-1}+1$

$3\left(1=\frac{1}{3}\sqrt{5x-1}\right)$

$(3)^2=\left(\sqrt{5x-1}\right)^2$

$9=5x-1$

$5x=10$

$x=2$

6) $-2=2(x-1)^{\frac{1}{3}}+4$

$-6=2(x-1)^{\frac{1}{3}}$

$(-3)^3=\left((x-1)^{\frac{1}{3}}\right)^3$

$-27=x-1$

$-26=x$

$x=-26$

Rational Equations

7) $\cancel{\frac{x}{3}}-\frac{1}{9x}\cancel{\frac{x}{3}}=\frac{5}{18}$

$\left(-\frac{1}{9x}=\frac{5}{18}\right)18x$

$-2=5x$

$x=-\frac{2}{5}$

* change from original packet

8) $\left(\frac{3}{x+1}-\frac{5x}{x^2-1}=0\right)(x+1)(x-1)$

$3(x-1)-5x=0$

$3x-3-5x=0$

$-2x-3=0$

$-2x=3$

$x=-\frac{3}{2}$

9) $\left(-\frac{1}{a}-\frac{3}{2a^2}=1\right)2a^2$

$-2a-3=2a^2$

$2a^2+2a+3=0$

$x=\frac{-(2)\pm\sqrt{(2)^2-4(2)(3)}}{2(2)}$

$x=\frac{-2\pm\sqrt{-20}}{4}$

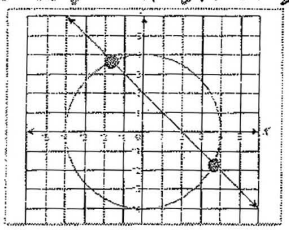
$=\frac{-2\pm 2i\sqrt{5}}{4}$ $x=\frac{-1\pm i\sqrt{5}}{2}$

Systems

Solve the system and state the number of solutions to the system.

* Mark location of solutions

10)



2 solutions

11) $\begin{cases} y = x^2 + 3 \\ y = x + 5 \end{cases}$

$x^2 + 3 = x + 5$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1 \quad x = 2$

$y = -1 + 5 = 4$

$y = 2 + 5 = 7$

$\{(-1,4),(2,7)\}$

Quadratics

Solve and describe the nature of the roots.

12) $2x^2 + 9x = -7$

$2x^2 + 9x + 7 = 0$

$(2x+7)(x+1) = 0$

$2x = -7 \quad x = -1$

$x = -\frac{7}{2}$

2 Real Rational Roots

13) $x^2 = -72$

$\sqrt{x^2} = \sqrt{-72}$

$x = \pm\sqrt{-36 \cdot 2}$

$x = \pm 6i\sqrt{2}$

2 Imaginary Roots

14) $x^2 - 6x = 1$

$x^2 - 6x - 1 = 0$

$x^2 - 6x + 9 = 1 + 9$

$\sqrt{(x-3)^2} = \pm\sqrt{10}$

$x - 3 = \pm\sqrt{10}$

$x = 3 \pm \sqrt{10}$

2 Real Irrational Roots

15) Find $r(0)$ for $r(x) = x^2 - 1$.

$(x+1)(x-1) = 0$

$x = -1 \quad x = 1$

$r(0) = \pm 1$

* Change #16, #17, #20 original problems

Mixed Practice

16) $\frac{x}{x^2 + 8x + 7} = \frac{1}{x+1} + 1$

$(x+1)(x+7)$

$x = x + 7 + x^2 + 8x + 7$

$0 = x^2 + 8x + 14$

$x^2 + 8x + 16 = -14 + 16$

$\sqrt{(x+4)^2} = \pm\sqrt{2}$

$x + 4 = \pm\sqrt{2}$

$x = -4 \pm \sqrt{2}$

17) $x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x + 5 = 0 \quad x - 2 = 0$

$x = -5 \quad x = 2$

$\{-5, 2\}$

18) $4\sqrt[3]{\frac{1}{4}x - 1} = -8$

$\left(\sqrt[3]{\frac{1}{4}x - 1}\right)^3 = (-2)^3$

$\frac{1}{4}x - 1 = -8$

$\frac{1}{4}x = -7$

$x = -28$

19) $\frac{3}{4}|x - 2| \geq 3$

$|x - 2| \geq 4$

$x - 2 \geq 4$ or $x - 2 \leq -4$

$x \geq 6$ or $x \leq -2$

20) $\frac{2x+1}{3} = \frac{1}{x}$ $3x$

$x(2x+1) = 3$

$2x^2 + x = 3$

$2x^2 + x - 3 = 0$

$(2x+3)(x-1) = 0$

$2x + 3 = 0 \quad x - 1 = 0$

$x = -\frac{3}{2} \quad x = 1$

21) $5x^2 - 10x + 5 = 0$

$5(x^2 - 2x + 1) = 0$

$5(x-1)^2 = 0$

$x - 1 = 0$

$x = 1$

SOL Review Topic 5: Graphs of Functions

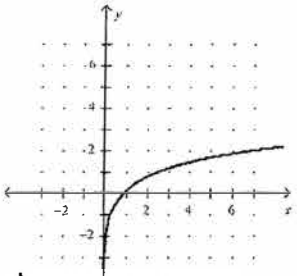
Logs, Exponentials, Absolute Value, Quadratics, Higher Order Polynomials, Cube, Cube Root, Square Roots, Rational Equations

(Increasing, Decreasing, Domain, Range, Transformations, Asymptotes, Inequalities)

Recognizing Graphs of Functions

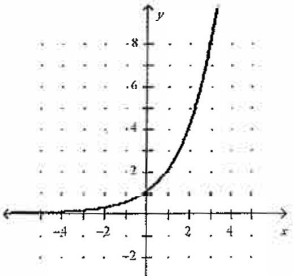
What is the name of the function show in each graph below? What is the equation of the graph?

1)



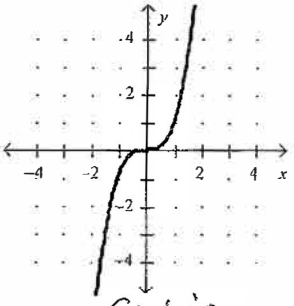
Logrithmic
$y = \log x$

2)



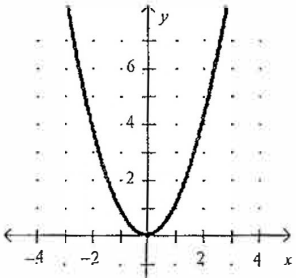
Exponential
$y = 2^x$

3)



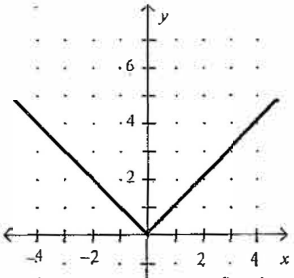
Cubic
$y = x^3$

4)



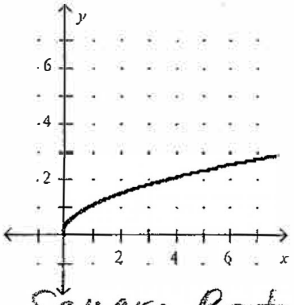
Quadratic
$y = x^2$

5)



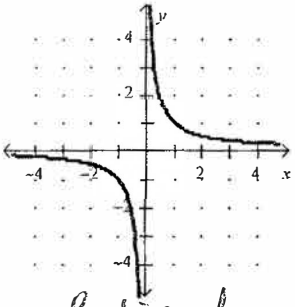
Absolute Value
$y = |x|$

6)



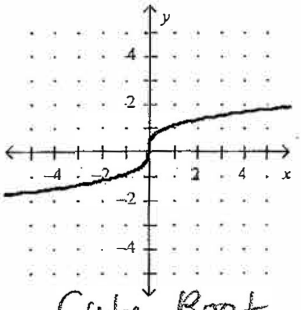
Square Root
$y = \sqrt{x}$

7)



Rational
$y = \frac{1}{x}$

8)



Cube Root
$y = \sqrt[3]{x}$

8) Which of the above graphs have a domain or all real numbers?

Exponential (2), Cubic (3), Quadric (4), Abs Value (5), Cube Root (8)

9) Which of the above functions have a range of all real numbers?

Logrithmic (1), Cubic (3), Cube Root (8)

10) Which of the above functions have asymptotes? What are the equations of the asymptotes?

Logrithmic	Exponential	Rational
$y = \log x$	$y = 2^x$	$y = \frac{1}{x}$
Asy: $x = 0$	Asy: $y = 0$	Asy: $x = 0$ and $y = 0$

Transformation Equations

For each of the following, name the function and the vertex (or pivot point). Then give the equation of the function after it has been shifted right by three and down 2.

11) $y = 2(x-3)^2 - 5$

Name: Quadratic

Vertex: $(3,-5)$

Translated Equation:

$y =$ $(6,-7)$

12) $y = (x-1)^3$

Name: Cubic

Vertex: $(1,0)$

Translated Equation:

$y =$ $(4,-2)$

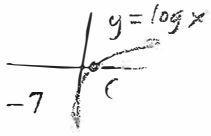
13) $y = |x-6|$

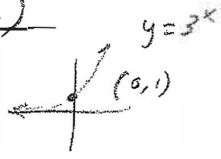
Name: Abs Value

Vertex: $(6,0)$

Translated Equation:

$y =$ $(9,-2)$





14) $y = \sqrt{x+4} + 15$

Name: Square Root

Vertex: $(-4,15)$

Translated Equation:

$y =$ $(-1,13)$

15) $y = \log(x+1) - 7$

Name: Log

Vertex: $(0,-7)$

Translated Equation:

$y =$ $(3,-9)$

16) $y = 3^x - 1$

Name: Exponential

Vertex: $(0,0)$

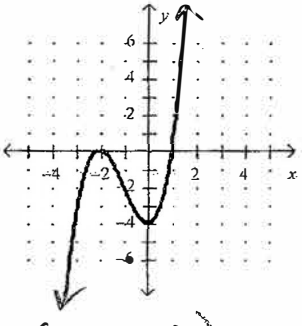
Translated Equation:

$y =$ $(3,-2)$

Domain, Range, Increasing Decreasing

For each of the following, determine the domain, range, intervals to which the function is increasing and decreasing, is sign of the leading coefficient and the end behavior.

17)



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing: $(-\infty,-2) \cup (0,\infty)$

Decreasing: $(-2,0)$

As $x \to \infty$, $f(x) \to$ $+\infty$

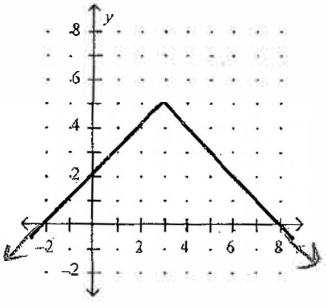
As $x \to -\infty$, $f(x) \to$ $-\infty$

Leading Coefficient: Positive

Factors: $(x+2)$, $(x-1)$

Possible Equation: $f(x) = (x-1)(x+2)^2$

18)



Domain: $(-\infty, \infty)$

Range: $(-\infty, 5]$

Increasing: $(-\infty, 3)$

Decreasing: $(3, \infty)$

As $x \to \infty$, $f(x) \to$ $-\infty$

As $x \to -\infty$, $f(x) \to$ $-\infty$

Leading Coefficient: negative

Asymptotes

Find all asymptotes of the following functions.

19) $y = \log(x-5)$

$\frac{\text{BIG}}{\text{Small}}$ = None

$\frac{\text{Same}}{\text{Same}}$ = Coeff

$\frac{\text{Small}}{\text{Big}}$ = $y=0$

$x=5$

Vertical: $x=5$

Horizontal: None

20) $y = 4^x - 1$

$y=1$

V: none

H: $y=1$

21) $y = \frac{x}{4x+1}$

$4x+1=0$

$4x=-1$

$x=-\frac{1}{4}$

$y=\frac{1}{4}$

V: $x=-\frac{1}{4}$

H: $y=\frac{1}{4}$

22) $y = \frac{1}{3x^2+3x-18}$

$0=(3x-6)(x+3)$

$3x-6=0$ $\quad x+3=0$

$3x=6$ $\quad x=-3$

$x=2$

V: $x=2,\ x=-3$

H: $y=0$

23) $y = \frac{x+4}{2x-6}$

$2x-6=0$

$2x=6$

$x=3$

V: $x=3$

H: $y=\frac{1}{2}$

24) $y = \frac{4}{x}+3$

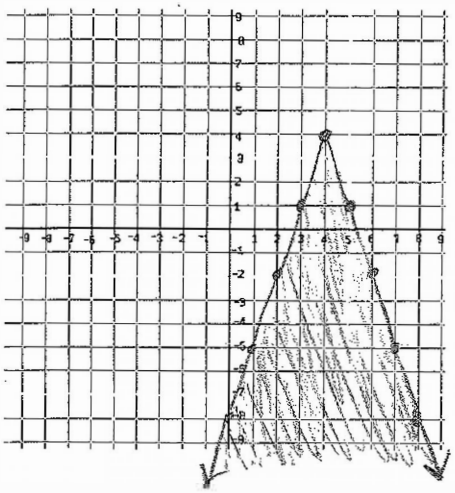
V: $x=0$

H: $y=0+3 \rightarrow y=3$

Inequalities

Graph the following inequality:

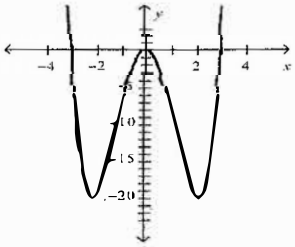
25) $y \le -3|x-4|+4$



Zeros: Find $f(0)$ for the following functions. Name the # of real and imaginary solutions & degree.

Remember $f(0)$'s = x-intercepts = zeros = solutions = roots.

26)



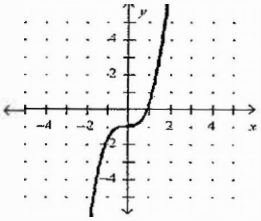
$f(0)$ = $-3, 0, 3$

# of Real Solutions = 3 (1 Double Root)

# of Imaginary Solutions = 0

Degree of Function: 4

27)



$f(0)$ = 1

# of Real Solutions = 1

# of Imaginary Solutions = 2

Degree of Function: 3

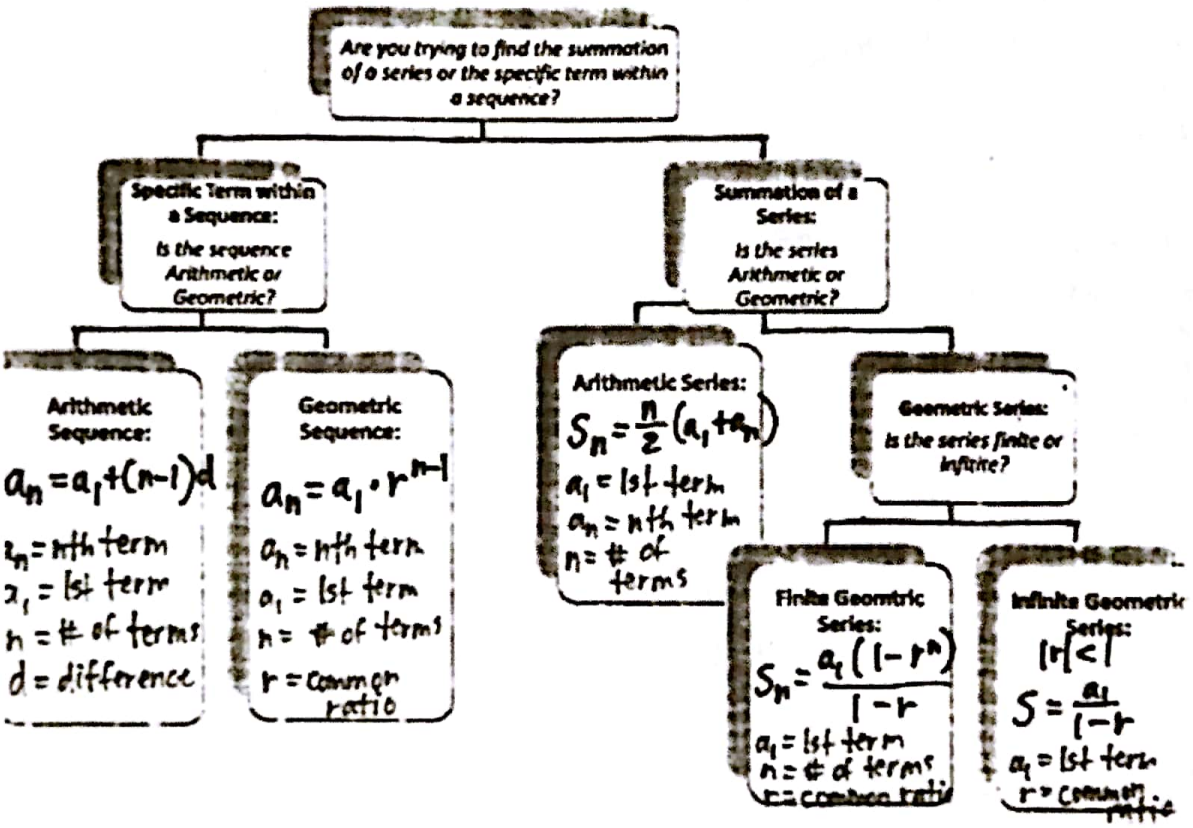
Algebra 2 Review Topic 6: Other!

Sequences and Series, Statistics, Composition of Functions, ~~Variation~~, Inverses, Properties

Sequences and Series

How do I know when to use each formula?

How do you know which formula to use for sequence and series problems?



Mixed Sequences and Series Practice

1) Find 1st 3 terms: $a_1 = 4,\ a_{n+1} = 2a_n + 1 \quad for\ n \geq 1$

a_n = current term
a_{n+1} = next term
a_{n-1} = previous term

next term = 2·current term + 1

4, 9, 19

2) Find a_{20} for $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

geometric

$r = \frac{a_2}{a_1} = \frac{1}{2}$

$a_n = a_1 \cdot r^{n-1}$

$a_{20} = 2\left(\frac{1}{2}\right)^{20-1}$

$a_{20} = \frac{1}{2^{18}}$

3) Find the 3 arithmetic means: 5, 3, 1, -1, -3

$-3 - 5 = -8$

$-8 \div 4 = -2$

4) Find the 17th term if $a_1 = -20$ & $d = 4$

arithmetic

$a_n = a_1 + (n-1)d$

$a_{17} = -20 + (17-1)(4)$

$a_{17} = 44$

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Mixed Sequences and Series Practice - Continued

5) 97 is the ___?___ th term of $-3, 1, 5, 9, \ldots$

arithmetic

$d = a_2 - a_1$

$d = 1-(-3) = 4$

$97 = -3 + (n-1)4$

$97 + 3 = (n-1)4$

$100 = 4(n-1)$

$25 = n-1$

$n = 26$

6) Find sum of $1-2+4-8+16\ldots$ to 15 terms $n=15$

geometric $r = \frac{a_2}{a_1} = \frac{-2}{1} = -2$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_{15} = \frac{1(1-(-2)^{15})}{1-(-2)} = 10923$

7) Find the sum of geometric series $a_1 = 10$, $a_n = 270$, $n = 4$

need r

$a_n = a_1 r^{n-1}$

$270 = 10 \cdot r^{4-1}$

$27 = r^3$

$r = \sqrt[3]{27} = 3$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_4 = \frac{10(1-3^4)}{1-3}$

$S_4 = 400$

8) Find the sum of $2, 1, \frac{1}{2}, \frac{1}{4} \ldots$ $r = \frac{a_2}{a_1} = \frac{1}{2}$

Infinite sum: $|r| < 1$

$S_\infty = \frac{a_1}{1-r} = \frac{2}{1-\frac{1}{2}} = 4$

Statistics 1 – Finding Regression Equation

9) Jean invested \$380 in stocks. Over the next 5 years, the value of her investment grew, as shown in the accompanying table. Write the regression equation for this set of data, rounding all values to *two decimal places*. Using this equation, find the value of her stock, to the *nearest dollar*, 10 years after her initial purchase.

Years Since Investment (x)	Value of Stock, in Dollars (y)
0	380
1	395
2	411
3	427
4	445
5	462

Stat Edit

Turn Stat Plot on

Zoom 9

Linear regression

Stat → Calc → LinReg(ax+b)

$y = 16.46x + 378.86$

$y = 16.46(10) + 378.86 = 543.46 \approx \$543$

Statistics 2 – Fundamental Counting Rule, Permutations, Combinations

10) In the next Olympics, the United States can enter four athletes in the diving competition. How many different teams of four divers can be selected from a group of nine divers?

$_9C_4 = 126$

11) Find the total number of different twelve-letter arrangements that can be formed using the letters in the word PENNSYLVANIA.

$_{12}P_{12} = 479{,}001{,}600$

12) A four-digit serial number is to be created from the digits 0 through 9. How many of these serial numbers can be created if 0 can not be the first digit, no digit may be repeated, and the last digit must be 5?

$\underline{8} \cdot \underline{8} \cdot \underline{7} \cdot \underline{1} = 448$

not 0 or 5

13) A multiple choice test has 10 questions where each question has 4answers. If you select one of the four answers for each question, how many different ways can you answer the questions?

$\underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4}$

$= 4^{10} = 1048576$

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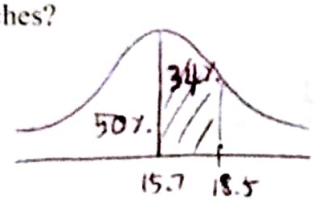
Statistics 3 – Normal Distribution and Z-Scores

14) The width of shark jaws are normally distributed with a mean of 15.7 and a standard deviation of 2.8 inches. What is the probability that a shark that you examine at random has a jaw width less than 18.5 inches?

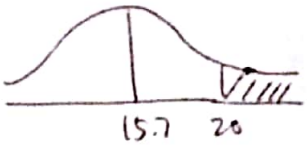


$z = \frac{18.5 - 15.7}{2.8} = 1$

50% + 34% = 84%

P(shark width < 18.5) = 0.84

15) What is the probability that a shark that you examine at random has a jaw greater than 20 inches?



$z = \frac{20 - 15.7}{2.8} = 1.54$ $\left(z = \frac{x - \mu}{\sigma} \right)$

Use table!

$P(z > 1.54) = 1 - P(z < 1.54)$

$= 1 - .9382 = 0.0618$

from table

approx. 6%

Composition of Functions

16) If $f(x) = \sqrt{x} + 1$ and $g(x) = x + 3$, then find $f \circ g$.

$f(g(x)) = f(x+3) = \sqrt{x+3} + 1$

17) If $f(x) = \frac{1}{x}$ and $g(x) = x^2 - x$, find $f(g(-1))$.

$f(g(-1)) = f(2) = \frac{1}{2}$

$g(-1) = (-1)^2 - (-1) = 2$

Inverses

1 to 1 function

18) Find the inverse of $y = \frac{1}{2}x - 2$.

switch x and y: $x = \frac{1}{2}y - 2$

$x + 2 = \frac{1}{2}y$

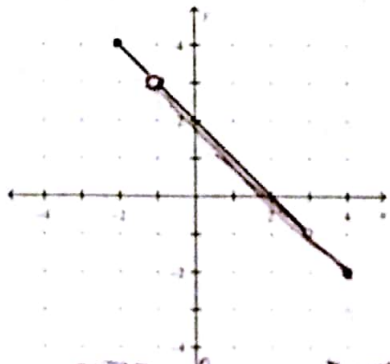
$2x + 4 = y$

$f^{-1}(x) = 2x + 4$

19) Is $y = x^2 - 2$ a one-to-one function?

Not a 1-to-1 function
Does not pass horizontal line test so its inverse is not a function

20) Graph the inverse of the line segment.



21) What is the range of the graphed line segment?

$(-1, 4]$

22) What is the domain of the inverse?

$(-1, 4]$

the same

original vs. inverse

(-2, 4) (4, -2)

(3, -1) (-1, 3)

swap x, y

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23) Graph $y = 10^x$ and the inverse of $y = 10^x$.

Inverse: $x = 10^y$

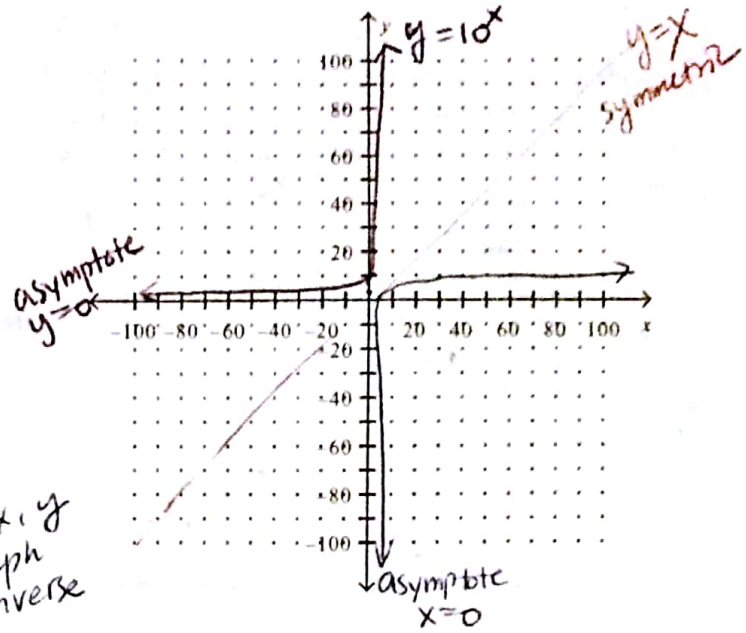
$y = \log_{10} x$

$f^{-1}(x) = \log_{10} x$

$y = 10^x$

x	y
0	1
1	10
2	100
-1	$\frac{1}{10}$
-2	$\frac{1}{100}$

switch x, y and graph inverse



Polynomial / Synthetic division

24) Divide: $\dfrac{-13x^2 + 4x^3 + 2x - 7}{x^2 + 3x - 2}$

$$\begin{array}{r} 4x - 25 \\ x^2+3x-2 \overline{\big) 4x^3 - 13x^2 + 2x - 7} \\ -(4x^3 + 12x^2 - 8x) \\ \hline -25x^2 + 10x - 7 \\ -(-25x^2 - 75x + 50) \\ \hline 85x - 57 \end{array}$$

$$\boxed{4x - 25 + \frac{85x - 57}{x^2 + 3x - 2}}$$

25) Divide: $\dfrac{2x^3 + 5x^2 + 9}{x + 3}$ $= 0$ ---- $x = -3$

-3	2	5	0	9
	↓	-6	3	9
	2	-1	3	0
	↑	↑	↑	
	x^2	x	#	

$$\boxed{2x^2 - x + 3}$$

Log/exponential equations base is always base

Convert each log expression into an exponential expression.

26) $\log_{12} 144 = 2$

$144 = 12^2$

27) $\log_4 \frac{1}{64} = -3$

$\frac{1}{64} = 4^{-3}$

28) $\log_{27} 3 = \frac{1}{3}$

$3 = 27^{\frac{1}{3}}$

Convert each exponential expression into a log expression.

29) $6^2 = 36$

$2 = \log_6 36$

30) $2^{-5} = \frac{1}{32}$

$-5 = \log_2 \frac{1}{32}$

31) $m^n = p$

$n = \log_m p$

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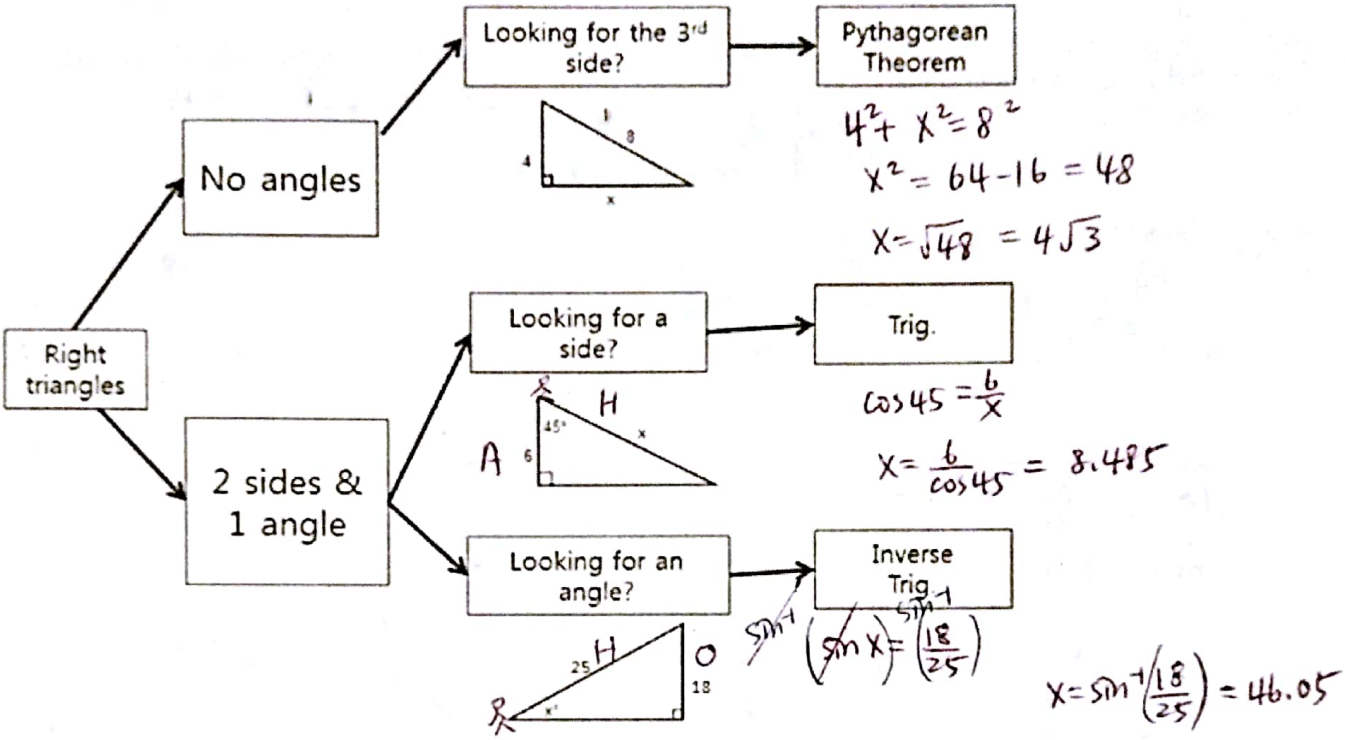
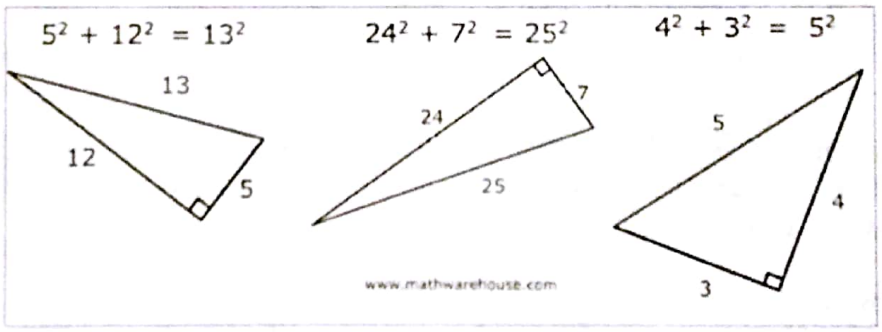
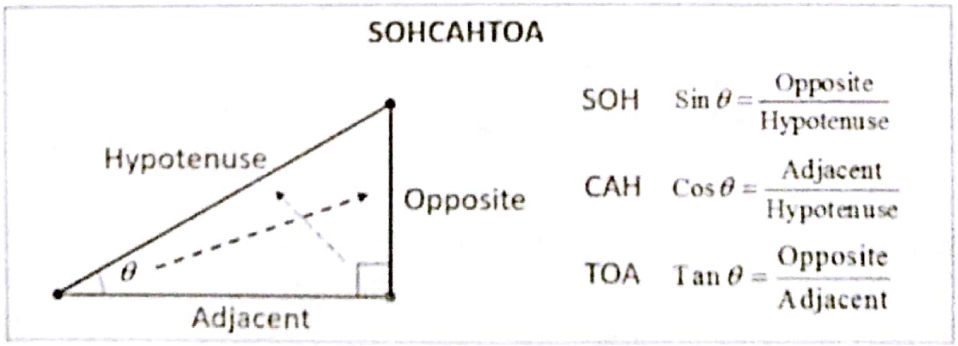
Topic 7 Trigonometry Review:

Pythagorean Theorem, SOH CAH TOA and word problems



Pythagorean Theorem example: set hypotenuse (longest side or across from right angle) to "c"





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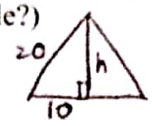
1. A side of an equilateral triangle is 20 cm long. What is the height/altitude of this triangle in the **simplest radical form**? (Do you see a right triangle?)

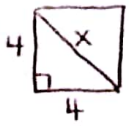


$h^2+10^2=20^2$

$h^2=300$

$h=\sqrt{300}=10\sqrt{3}$

2. A side of a square is 4 cm long. What is the diagonal of the square in the **simplest radical form**? (Do you see a right triangle?)



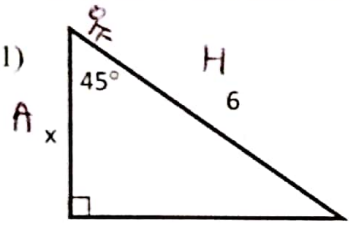
$4^2+4^2=x^2$

$x^2=32$

$x=\sqrt{32}=4\sqrt{2}$

3. Solve for x. Round to the tenth.

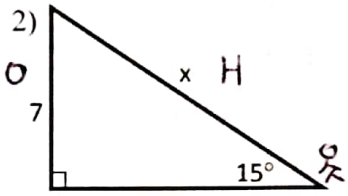
1)



$\cos 45=\frac{x}{6}$ $x=6\cos 45$

$=4.2$

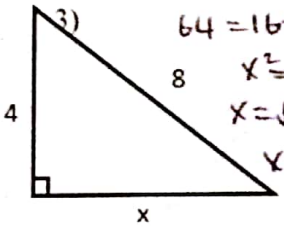
2)



$\sin 15=\frac{7}{x}$

$x=\frac{7}{\sin 15}=27.0$

3)



$8^2=4^2+x^2$

$64=16+x^2$

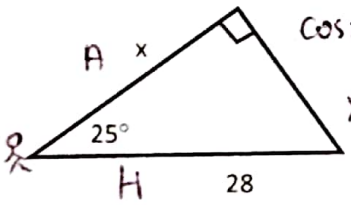
$x^2=48$

$x=\sqrt{48}$

$x=4\sqrt{3}$

4. Solve for x.Round to the tenth.

1)

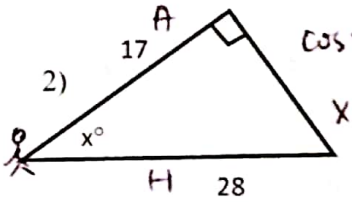


$\cos 25=\frac{x}{28}$

$x=28\cos 25$

$x=25.4$

2)



$\cos x=\frac{17}{28}$

$x=\cos^{-1}\frac{17}{28}=52.6$

diagonal → $c^2=a^2+b^2$

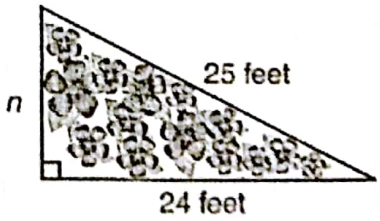
5. Which window with the following dimensions is too small to allow a 35-inch piece of glass to fit through it?

A. 28 ×45 inches $c^2=28^2+45^2$ $c=53$ B. 16 ×33 inches $c^2=16^2+33^2$ $c=36.7$

C. 20 ×28 inches $c^2=20^2+28^2$ $c=34.4$ D. 40 ×42 inches $c^2=40^2+42^2$ $c=58$

diagonal is shorter than 35

6. Stephen is planning a right triangular garden. He marked two sides that measure 24 feet and 25 feet. He wants to know the perimeter of the garden so that he can find out how many bricks he should buy at the store. Find the perimeter of the garden.



$n^2+24^2=25^2$

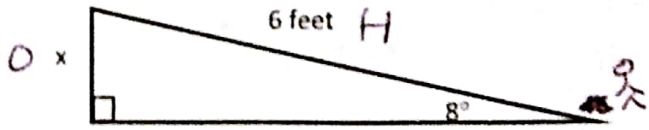
$n^2=625-576=49$

$n=7$

$7+24+25$

$=\boxed{56}$

7. The trainer adjusted a 6-**foot** long bench press so that the angle of elevation was 8°. How many **inches** did the trainer raise the bench press?



$\sin 8°=\frac{x}{6}$

$x=6\sin 8°=.8350$ ft

times 12 10.0 inches

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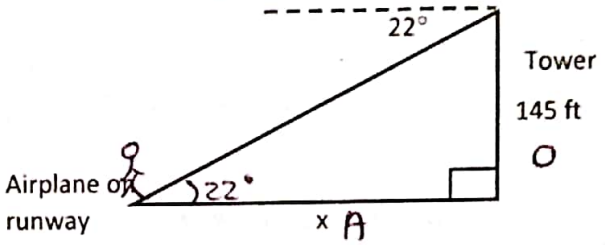
8. From the top of a 145-foot high tower, an air traffic controller observes an airplane on the runway at an angle of depression of 22°. How far from the base of the tower is the airplane?

$\tan 22 = \frac{145}{x}$

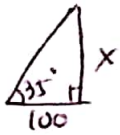
$x = \frac{145}{\tan 22} = \boxed{358.9 \text{ ft}}$



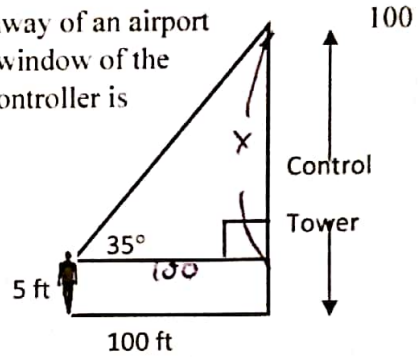
9. Chelsea whose eyes are 5 feet above the ground is standing on the runway of an airport feet from the control tower. She observes an air traffic controller at the window of the control tower. The angle of elevation from the person to the air traffic controller is 35°. How tall is the control tower?

$\tan 35 = \frac{x}{100}$

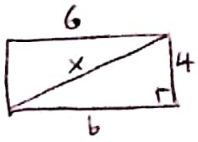
$x = 100 \tan 35 = 70$



tower height = eye height + x = $\boxed{75 \text{ ft}}$



10. Liz is building a rectangular gate. The dimensions of the gate are 6 feet high and 4 feet wide. She wants to fasten a thin brace diagonally at the corners to keep the gate sturdy. Approximately, how long is the brace?



$6^2 + 4^2 = x^2$

$36 + 16 = x^2$

$x^2 = 52$

$x = \sqrt{52}$

$4 \quad 13$

$2 \quad 2$

$\boxed{x = 2\sqrt{13}}$

$a^2 + b^2 = c^2$

11. Rosemary is cutting 3 wooden sticks to build part of a kite frame. The part she is building must be a right triangle. Which choice below could be the lengths, in inches, of the sticks Rosemary cut? Choose all that apply.

A. 4, 5, 6 $\quad 4^2 + 5^2 \neq 6^2$

B. 4, 3, 5 $\quad 3^2 + 4^2 = 5^2$

C. 10, 15, 12 $\quad 10^2 + 12^2 \neq 15^2$

D. 12, 13, 5 $\quad 5^2 + 12^2 = 13^2$

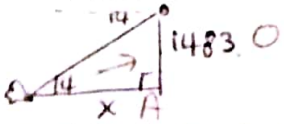
E. $\sqrt{7}, \sqrt{5}, 4$ $\quad (\sqrt{7})^2 + (\sqrt{5})^2 = 12$, $4^2 = 16$

F. $\sqrt{3}, \sqrt{6}, 3$ $\quad (\sqrt{3})^2 + (\sqrt{6})^2 = 9$, $3^2 = 9$

G. 9, 40, 41 $\quad 9^2 + 40^2 = 1681$, $41^2 = 1681$

H. $\sqrt{5}, 2\sqrt{2}, 13$ $\quad (\sqrt{5})^2 + (2\sqrt{2})^2 = 13$, $13^2 = 169$

12. The angle of depression of an object on the ground is 14° from the top of the tallest building in the world, one of Petronas towers in Malaysia, which is 1,483 feet high. What is the distance from the object to the base of the tower to the nearest foot?

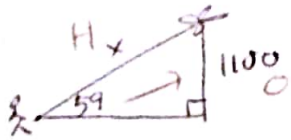


$\tan 14 = \frac{1483}{x}$

$x = \frac{1483}{\tan 14} = 5947.988\ldots$

$\boxed{5948 \text{ ft}}$

13. You are at the air show in Virginia Beach. You are looking up at a British Harrier Jet at an angle of elevation of 59°. If the plane is hovering 1100 ft above the water, how far are you from the jet (direct distance)? Round to the nearest foot.



$\sin 59 = \frac{1100}{x}$

$x = \frac{1100}{\sin 59} = 1283.2967\ldots$

$\boxed{1283 \text{ ft}}$

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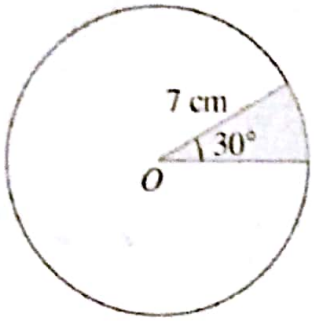
Finding Arc Length & Area of Sector



$$Arc\ Length = \frac{\theta}{360°} \times 2\pi r$$

$$= \frac{30°}{360°} \times 2 \times \frac{22}{7} \times 7cm$$

$$= 3.667\ cm$$

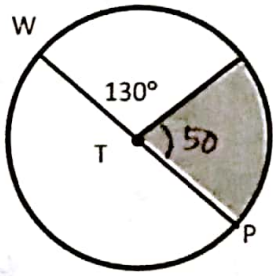
$$Area\ of\ Sector = \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{30°}{360°} \times \frac{22}{7} \times 7^2$$

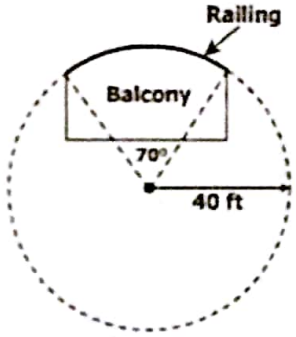
$$= 12.83\ cm^2$$

diameter radius=18

1. Given circle T with WP = 36 cm. Calculate the exact **area** of the shaded sector.

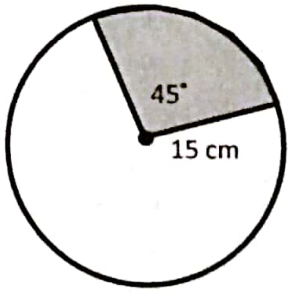


$$\pi r^2 \frac{\theta}{360} = \pi(18)^2\left(\frac{50}{360}\right) = \boxed{141.37}$$



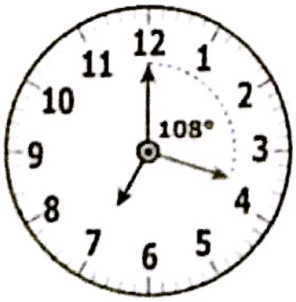
2. Find the length of the balcony. arc

$$2\pi r\left(\frac{\theta}{360}\right) = 2\pi(40)\frac{70}{360} = \boxed{48.87}$$



3. Calculate the area of the shaded sector, to the nearest tenth.

$$\pi r^2\left(\frac{\theta}{360}\right) = \pi(15^2)\frac{45}{360} = \boxed{88.36}$$



4. The minute hand on a clock is 10 centimeters long and travels through an arc of 108° every 18 minutes. Find the measure of the length of the arc the minute hand travels through during this 18-minute period.

$$2\pi r\left(\frac{\theta}{360}\right) = 2\pi(10)\left(\frac{108}{360}\right) = \boxed{18.85}$$

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