IB Extended Math 2019 Summer Review

Name: _____ Grade: _____

- You are expected to know all topics from Algebra 2 and trigonometry from Geometry.
- This packet is optional **due on 9/6**. It will be worth **4 classwork grades** in September.
- The answer keys will be on the school website & IB website or **tinyurl.com/jrtlee**
- You will have several quizzes on these topics in the first marking period.
- If you have trouble reviewing the topics over the summer, use resources such as Khan academy, IXL, and youtube.

- You will be given homework every class and the homework will be checked every class in IB Extended Math.
- There is no credit for late homework and it **DOES AFFECT** your grade!

The topics of IB extended Math are:

1) Reviewing and extending algebra 2 topics (3 months)
2) Trigonometry (3 months)
3) Probabilities and statistics (2 weeks)
4) Sequences and series (2 weeks)
5) Exponentials and logarithms (1 month)
6) Polar coordinates (1 week)

Algebra 2 is crucial for you to be successful in the upcoming advanced math classes including IB Extended Math and SAT/ACT tests. If you don't think your algebra 2 skills are strong, please spend time over the summer. You will be expected to understand everything in this packet. You learned everything in algebra 1, geometry, and algebra 2 classes!

Table of Contents:

Algebra 2 Review Topic 1: Factoring and Solving Quadratics

7 Factoring Rules

Rule #1 - GCF

1) $3a^2bc^5 - 9a^3bc^6 + 12a^2b^3c^6$

Rule #2 – Difference of Perfect Squares

2) $9x^2 - 49y^2$

3) $100x^2 - 100$

Rule #3 – Trinomial with 1 as a Leading Coefficient

4) $a^2 + a - 20$

5) $3x^2 - 18x + 24$

Rule #4 – Sum/Difference of Perfect Cubes

6) $x^3 - 125y^3$

7) $64a^3 + 8y^3$

Rule #5 – Perfect Square Trinomials

8) $9x^2 + 30x + 25$

9) $36y^2 - 84y + 49$

Rule #6 – Factoring by Grouping

10) $a^2x - b^2x + a^2y - b^2y$

11) $4x^3 + 4x^2 - 6x - 6$

Rule #7 – Trinomial with Leading Coefficient >1

12) $3y^2 + 5y + 2$

13) $6x^2 - 5x - 6$

What if it can't be factored?

14) $x^2 + 25y^2$

Solving Quadratics

What are the different ways to solve quadratics?

#1 – Solve by Factoring

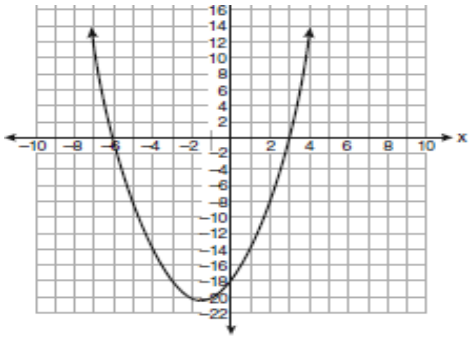
19) $w^2 - 8w - 9 = 0$

20) $25p^2 - 36 = 0$

21) $3x^2 = 16x - 5$

#2 – Solve by Graphing

22)



#3 – Solve by Completing the Square

23) $p^2 - 12p + 36$

#4 – Solve by the Quadratic Formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

24) $x^2 - 6x + 21 = 0$

25) $x + 2x^2 + 1 = -1 - x$

How do I know which way to solve?
Try to solve by factoring first, if you can't solve by factoring use the quadratic formula or solve by completing the square.

Solving Quadratics Mixed Practice

26) $x^2 + 4x = 3$

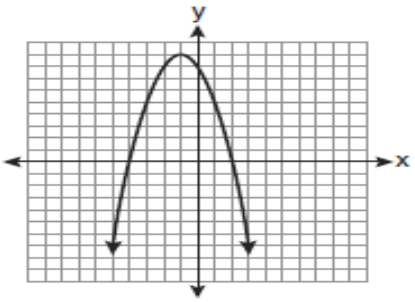
27) $2x^2 + 5x = 3$

28) $x^2 = 5x$

29) $x^2 + 3x - 40 = 0$

30) $3x^2 - 5x + 2 = 0$

31)



Describe the Nature of the Roots of a Quadratic

A) 2 Real Rational Roots
B) 1 Real Double Root
C) 2 Real Irrational Roots
D) 2 Imaginary Roots

What are the nature of the roots of the following quadratics with roots?

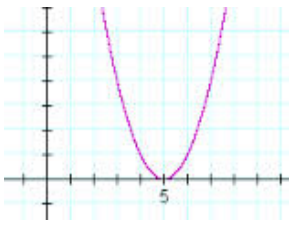
32) $\left\{\pm\frac{1}{3}\right\}$

33) $\left\{\pm\frac{1}{3}i\right\}$

34) $\{0, -5\}$

35) $\{2 \pm \sqrt{5}\}$

36)



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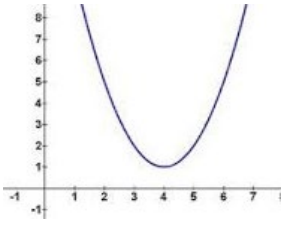


Algebra 2 Review Topic 2: Rational Expressions and Equations

Simplifying Rational Expressions

Factor and Reduce!

1) $\dfrac{2x^2+3x-2}{2x^2-8}$

Multiplying and Dividing Rational Expressions

2) $\dfrac{x^2-1}{1+x} \div \dfrac{2x^2-5x+3}{3-2x}$

3) $\dfrac{\dfrac{6x+12}{5x}}{\dfrac{x+2}{10x}}$

Adding and Subtracting Rational Expressions

YOU NEED A COMMON DENOMINATOR WHENEVER YOU ARE ADDING OR SUBTRACTING FRACTIONS!

Do not cancel on top and bottom! Get a common denominator & then add or subtract the numerators.

4) $\dfrac{8}{x-2} - \dfrac{4}{x+2}$

5) $\dfrac{c+d}{2cd^2} + \dfrac{d-c}{6c^2d}$

Solving Rational Equations

Multiply both sides of the equation by the LCD then cancel the fractions and simplify.

6) $\dfrac{x+1}{3(x-2)} = \dfrac{5x}{6} + \dfrac{1}{x-2}$

7) $\dfrac{4}{u+2} - \dfrac{2}{u-2} = \dfrac{u}{u^2-4}$

Mixed Practice Simplifying and Adding and Subtracting Rational Expressions

8) $\dfrac{x+7}{x-2} - \dfrac{4}{x^2+x-6} = 1$

9) $\dfrac{x^2-7x+12}{x^2-x-6} \cdot \dfrac{x^2+7x+10}{x^2+x-20}$

10) $\dfrac{2-5x}{x-9} + \dfrac{4x-5}{9-x}$

11) $\dfrac{x^2-9}{4} \div \dfrac{3-x}{8}$

Algebra 2 Review Topic 3: Simplifying Radicals and Complex Numbers

Simplifying Radicals

1) $\sqrt{200a^2b^2c^{11}}$ 2) $\sqrt[3]{54}$ 3) $\sqrt[4]{16a^5b^{11}}$ 4) $\sqrt[3]{-8a^4b^9c^{11}}$

Operations with Radicals

5) $\sqrt{72x^3y}\cdot\sqrt{50xy^3}$ 6) $(2-\sqrt{3})(2+\sqrt{3})$ 7) $\sqrt[3]{24}-\sqrt[3]{81}+\sqrt[3]{3}$

8) $\dfrac{1}{\sqrt[4]{2}}$ 9) $\sqrt{18x}-\sqrt{8x}$ 10) $\dfrac{2}{3-\sqrt{2}}$

Rational Exponents

Express the following in exponential form:

11) $\sqrt[3]{(3x)^2}$ 12) $\sqrt{7}$ 13) $\sqrt[4]{4a^5}$ 14) $\sqrt[4]{(4a)^5}$

Express the following in radical form:

15) $r^{-\frac{1}{3}}$ 17) $a^{\frac{2}{3}}b^{\frac{1}{2}}$

Powers of *i*

18) $i=$ $i^2=$ $i^3=$ $i^4=$
$i^5=$ $i^6=$ $i^7=$ $i^8=$

19) i^{33} 20) i^{102}

Imaginary Numbers

21) $\sqrt{-50}$ 22) $\sqrt{-3}\cdot\sqrt{-3}$ 23) $\dfrac{2}{2-3i}$

Operations with Complex Numbers

24) $(2+i)(4-3i)$

25) $(1-5i)-(4+i)+i$

Solving Radical and Rational Exponent Equations

26) $(5a-5)^{\frac{1}{3}}+1=3$

27) $\sqrt{x-1}+2=-1$

28) $\sqrt[5]{\frac{1}{5}x-7}-1=-2$

Mixed Practice with Radicals and Complex Numbers
SIMPLIFY OR SOLVE:

29) $-2\sqrt[4]{4x-12}+1=-3$

30) $x^2+49=0$

31) i^{93}

32) $\sqrt{-7}\cdot\sqrt{-7}$

33) $4\sqrt[4]{16p^4q^9}$

34) $\sqrt{24p^3q^{12}r^{24}}$

35) $\left(2+\sqrt{5}\right)\left(3-2\sqrt{5}\right)$

36) $3\sqrt{75}-12\sqrt{3}-\sqrt{18}+5\sqrt{8}$

37) $3\left(x-2\right)^{\frac{3}{4}}+1=25$

Algebra 2 Review Topic 4: Solving Equations and Inequalities

Absolute Value Equations, Rational Equations, Radical Equations, Quadratics, Systems, Inequalities

Absolute Value Equations

1) $3|2x-7|+2=17$

2) $|9+4x|=5x+18$

Absolute Value Inequalities

3) $\frac{1}{2}|x-8|-3\le 16$

4) $|x-3|-2\le 1$

Radical and Rational Exponent Equations

5) $2=\frac{1}{3}\sqrt{5x-1}+1$

6) $-2=2(x-1)^{\frac{1}{3}}+4$

Rational Equations

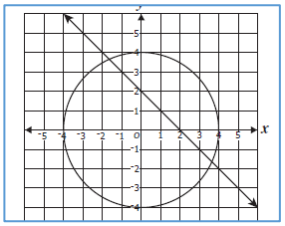
7) $\frac{x}{3}-\frac{1}{9x}-\frac{x}{3}=\frac{5}{18}$

8) $\frac{3}{x+1}-\frac{5x}{x^2-1}=2$

9) $-\frac{1}{a}-\frac{3}{2a^2}=1$

Systems

Solve the system and state the number of solutions to the system.

10) 

11) $\begin{cases} y = x^2 + 3 \\ y = x + 5 \end{cases}$

Quadratics

Solve and describe the nature of the roots.

12) $2x^2 + 9x = -7$

13) $x^2 = -72$

14) $x^2 - 6x = 1$

15) Find $r(0)$ for $r(x) = x^2 - 1$.

Mixed Practice

16) $\dfrac{x}{x^2 + 8x + 7} = \dfrac{1}{x+1} + 1$

17) $x^2 + 3x - 10 = 0$

18) $4\sqrt[3]{\dfrac{1}{4}x - 1} = -8$

19) $\dfrac{3}{4}|x - 2| \geq 3$

20) $\dfrac{2x+1}{3} = \dfrac{1}{x}$

21) $5x^2 - 10x + 5 = 0$

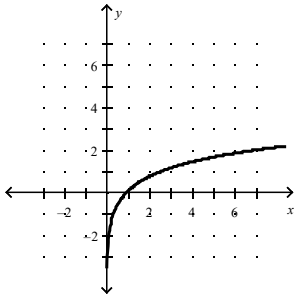
Algebra 2 Reveiw Topic 5: Graphs of Functions

Logs, Exponentials, Absolute Value, Quadratics, Higher Order Polynomials, Cube, Cube Root, Square Roots, Rational Equations
(Increasing, Decreasing, Domain, Range, Transformations, Asymptotes, Inequalities)

Recognizing Graphs of Functions

What is the name of the function show in each graph below? What is the equation of the graph?

1)



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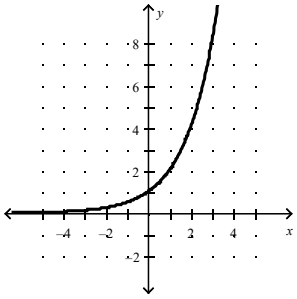

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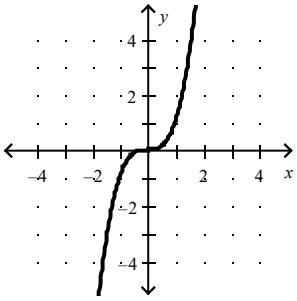

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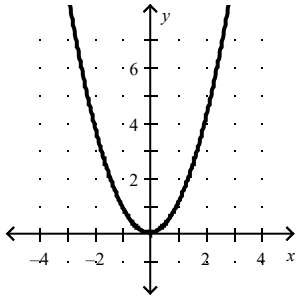

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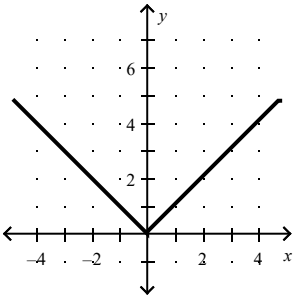

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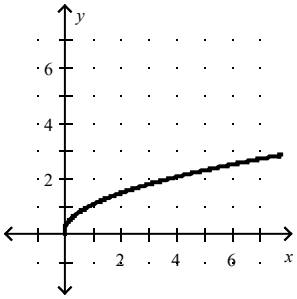

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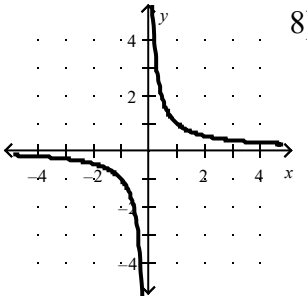

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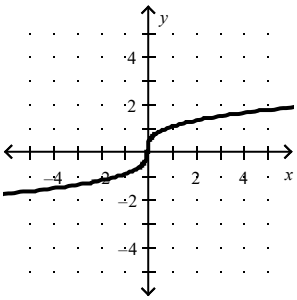


8) Which of the above graphs have a domain or all real numbers?

9) Which of the above functions have a range of all real numbers?

10) Which of the above functions have asymptotes? What are the equations of the asymptotes?

Transformation Equations

For each of the following, name the function and the vertex (or pivot point). Then give the equation of the function after it has been shifted right by three and down 2.

11) $y = 2(x-3)^2 - 5$

Name: _____

Vertex: _____

Translated Equation:

$y =$ _____

12) $y = (x-1)^3$

Name: _____

Vertex: _____

Translated Equation:

$y =$ _____

13) $y = |x-6|$

Name: _____

Vertex: _____

Translated Equation:

$y =$ _____

14) $y = \sqrt{x+4} + 15$

Name: _____

Vertex: _____

Translated Equation:

$y =$ _____

15) $y = \log(x+1) - 7$

Name: _____

Vertex: _____

Translated Equation:

$y =$ _____

16) $y = 3^x - 1$

Name: _____

Vertex: _____

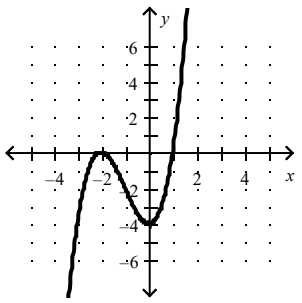
Translated Equation:

$y =$ _____

Domain, Range, Increasing Decreasing

For each of the following, determine the domain, range, intervals to which the function is increasing and decreasing, is sign of the leading coefficient and the end behavior.

17)



Domain: _____

Range: _____

Increasing: _____

Decreasing: _____

As $x \to \infty, f(x) \to$ ____

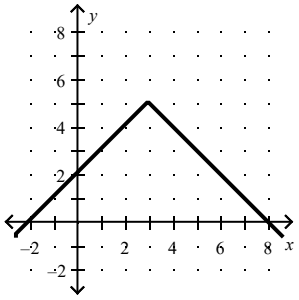
As $x \to -\infty, f(x) \to$ ____

Leading Coefficient: _____

Factors: _____

Possible Equation: _____

18)



Domain: _____

Range: _____

Increasing: _____

Decreasing: _____

As $x \to \infty, f(x) \to$ ____

As $x \to -\infty, f(x) \to$ ____

Leading Coefficient: _____

Asymptotes

Find all asymptotes of the following functions.

19) $y = \log(x-5)$

20) $y = 4^x - 1$

21) $y = \dfrac{x}{4x+1}$

22) $y = \dfrac{1}{3x^2+3x-18}$

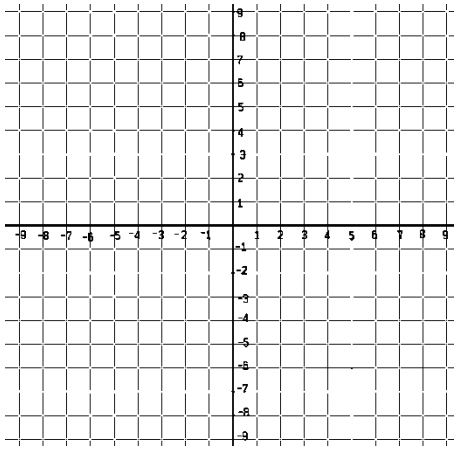
23) $y = \dfrac{x+4}{2x-6}$

24) $y = \dfrac{4}{x} + 3$

Inequalities

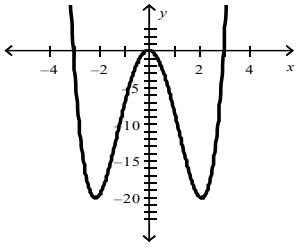
Graph the following inequality:

25) $y \le -3|x-4|+4$



Zeros: Find $f(0)$ for the following functions. Name the # of real and imaginary solutions & degree. Remember that x-intercepts = zeros = solutions = roots.

26)



$f(0) =$ _____

# of Real Solutions = _____

# of Imaginary Solutions = _____

Degree of Function: _____

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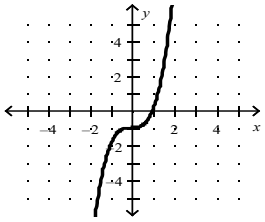


$f(0) =$ _____

# of Real Solutions = _____

# of Imaginary Solutions = _____

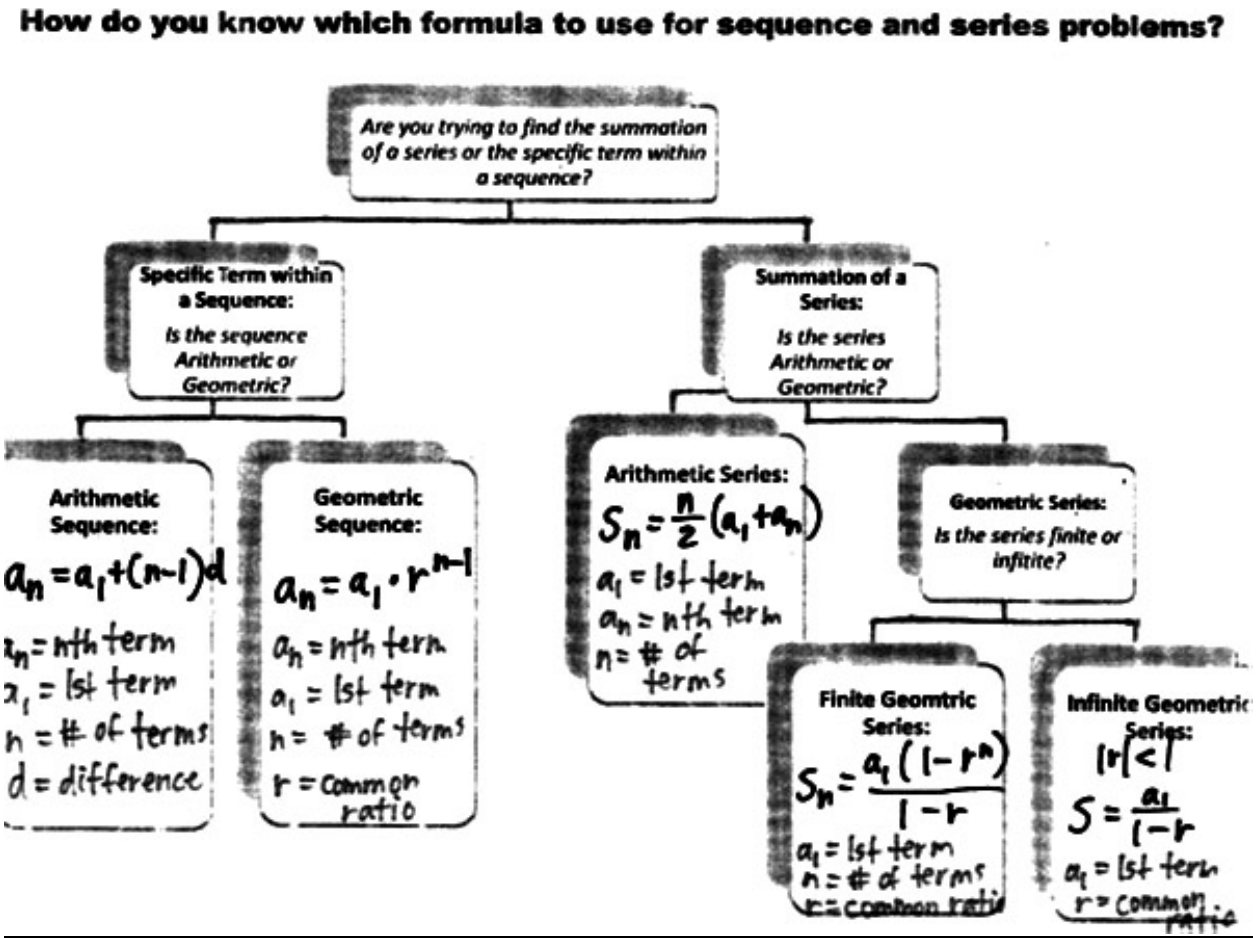
Degree of Function: _____

Algebra 2 Review Topic 6: Other!

Sequences and Series, Statistics, Composition of Functions, Variation, Inverses, Properties

Sequences and Series

How do I know when to use each formula?



Mixed Sequences and Series Practice

1) Find 1st 3 terms: $a_1 = 4,\ a_{n+1} = 2a_n + 1 \quad for\ n \geq 1$

2) Find a_{20} for $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

3) Find the 3 arithmetic means: 5, ___, ___, ___, -3

4) Find the 17th term if $a_1 = -20$ & $d = 4$

Mixed Sequences and Series Practice - Continued

5) 97 is the ___?__ th term of –3, 1, 5, 9, …

6) Find sum of $1-2+4-8+16...$ to 15 terms

7) Find the sum of geometric series $a_1 = 10,\ a_n = 270,\ n = 4$

8) Find the sum of $2,1,\frac{1}{2},\frac{1}{4}....$

Statistics 1 – Finding Regression Equation

9) Jean invested $380 in stocks. Over the next 5 years, the value of her investment grew, as shown in the accompanying table. Write the regression equation for this set of data, rounding all values to *two decimal places*. Using this equation, find the value of her stock, to the *nearest dollar*, 10 years after her initial purchase.

Years Since Investment (x)	Value of Stock, in Dollars (y)
0	380
1	395
2	411
3	427
4	445
5	462

Statistics 2 – Fundamental Counting Rule, Permutations, Combinations

10) In the next Olympics, the United States can enter four athletes in the diving competition. How many different teams of four divers can be selected from a group of nine divers?

11) Find the total number of different twelve-letter arrangements that can be formed using the letters in the word PENNSYLVANIA.

12) A four-digit serial number is to be created from the digits 0 through 9. How many of these serial numbers can be created if 0 can not be the first digit, no digit may be repeated, and the last digit must be 5?

13) A multiple choice test has 10 questions where each question has 4answers. If you select one of the four answers for each question, how many different ways can you answer the questions?

Statistics 3 – Normal Distribution and Z-Scores

14) The width of shark jaws are normally distributed with a mean of 15.7 and a standard deviation of 2.8 inches. What is the probability that a shark that you examine at random has a jaw width less than 18.5 inches?

15) What is the probability that a shark that you examine at random has a jaw greater than 20 inches?

Composition of Functions

16) If $f(x)=\sqrt{x}+1$ and $g(x)=x+3$, then find $f \circ g$.

17) If $f(x)=\frac{1}{x}$ and $g(x)=x^2-x$, find $f(g(-1))$.

Inverses

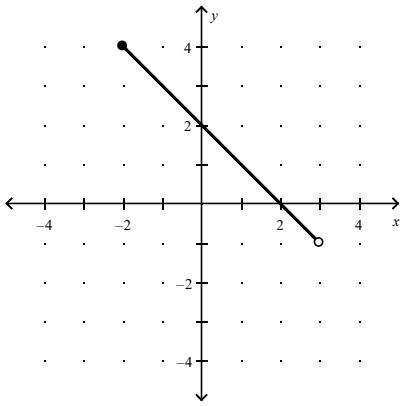
18) Find the inverse of $y=\frac{1}{2}x-2$.

19) Is $y=x^2-2$ a one-to-one function?

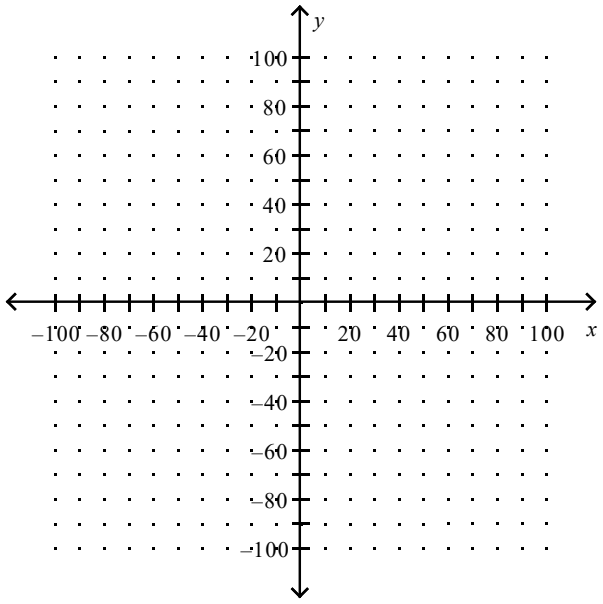
20) Graph the inverse of the line segment.

21) What is the range of the graphed line segment?

22) What is the domain of the inverse?



23) Graph $y = 10^x$ and the inverse of $y = 10^x$.



Polynomial / Synthetic division

24) Divide: $\dfrac{-13x^2 + 4x^3 + 2x - 7}{x^2 + 3x - 2}$

25) Divide: $\dfrac{2x^3 + 5x^2 + 9}{x + 3}$

Log/exponential equations

Convert each log expression into an exponential expression.

26) $\log_{12} 144 = 2$

27) $\log_4 \dfrac{1}{64} = -3$

28) $\log_{27} 3 = \dfrac{1}{3}$

Convert each exponential expression into a log expression.

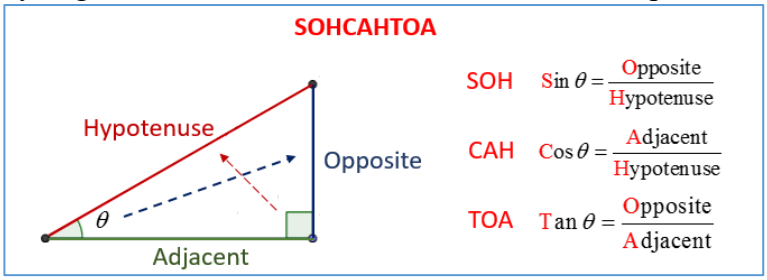
29) $6^2 = 36$

30) $2^{-5} = \dfrac{1}{32}$

31) $m^n = p$

Topic 7 Trigonometry Review:

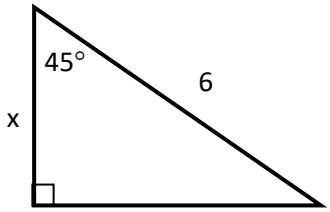
Pythagorean Theorem, SOH CAH TOA and word problems

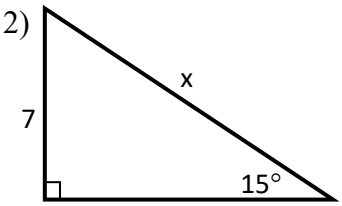


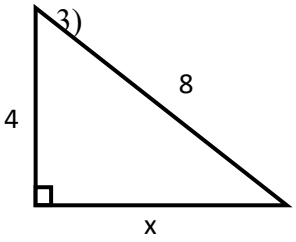
1. A side of an equilateral triangle is 20 cm long. What is the height/altitude of this triangle in the **simplest radical form**? (Do you see a right triangle?)

2. A side of a square is 4 cm long. What is the diagonal of the square in the **simplest radical form**? (Do you see a right triangle?)

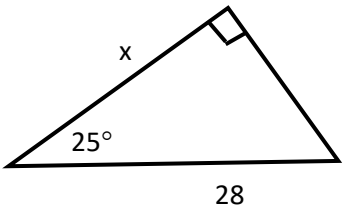
3. Solve for x. Round to the tenth.

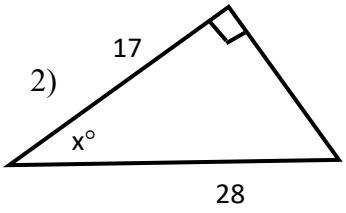
1) 

2) 

3) 

4. Solve for x.Round to the tenth.

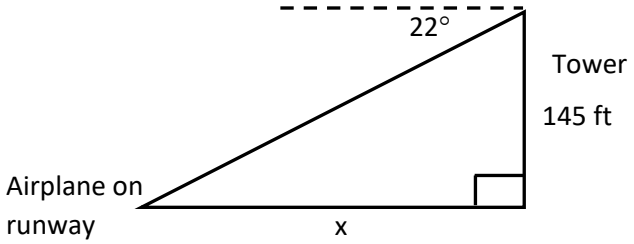
1) 

2) 

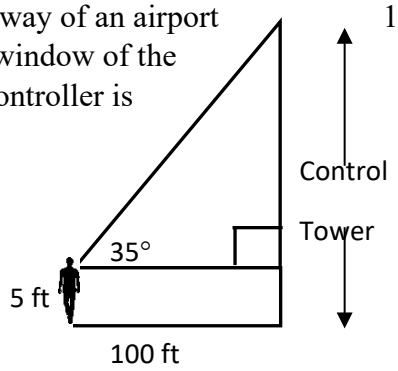
5. Which window with the following dimensions is too small to allow a 35-inch piece of glass to fit through it?

A. 28 ×45 inches
B. 16 ×33 inches
C. 20 ×28 inches
D. 40 ×42 inches

6. From the top of a 145-foot high tower, an air traffic controller observes an airplane on the runway at an angle of depression of 22°. How far from the base of the tower is the airplane?



7. Chelsea whose eyes are 5 feet above the ground is standing on the runway of an airport 100 feet from the control tower. She observes an air traffic controller at the window of the control tower. The angle of elevation from the person to the air traffic controller is 35°. How tall is the control tower?



8. Liz is building a rectangular gate. The dimensions of the gate are 6 feet high and 4 feet wide. She wants to fasten a thin brace diagonally at the corners to keep the gate sturdy. Approximately, how long is the brace?

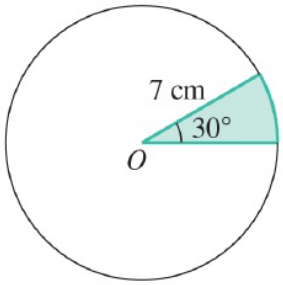
9. Rosemary is cutting 3 wooden sticks to build part of a kite frame. The part she is building must be a right triangle. Which choice below could be the lengths, in inches, of the sticks Rosemary cut? Choose all that apply.

A. 4, 5, 6 B. 4, 3, 5 C. 10, 15, 12 D. 12, 13, 5

E. $\sqrt{7}, \sqrt{5}, 4$ F. $\sqrt{3}, \sqrt{6}, 3$ G. 9, 40, 41 H. $\sqrt{5}, 2\sqrt{2}, 13$

10. The angle of depression of an object on the ground is 14° from the top of the tallest building in the world, one of Petronas towers in Malaysia, which is 1,483 feet high. What is the distance from the object to the base of the tower to the nearest foot?

Finding Arc Length & Area of Sector



$$Arc\ Length = \frac{\theta}{360°} \times 2\pi r$$

$$= \frac{30°}{360°} \times 2 \times \frac{22}{7} \times 7cm$$

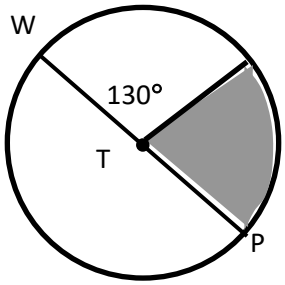
$$= 3.667\ cm$$

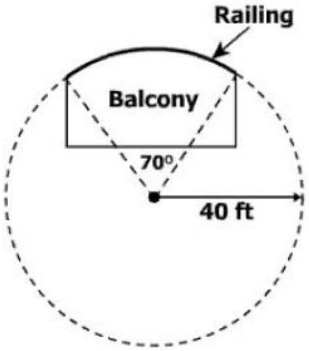
$$Area\ of\ Sector = \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{30°}{360°} \times \frac{22}{7} \times 7^2$$

$$= 12.83\ cm^2$$

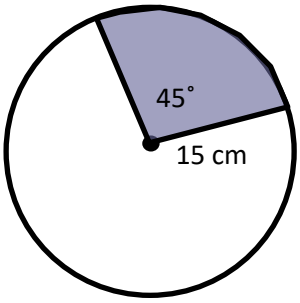
1. Given circle T with WP = 36 cm. Calculate the exact **area** of the shaded sector.

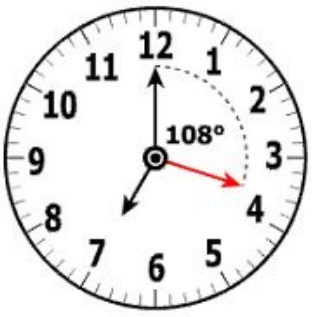




2. Find the length of the balcony.

3. Calculate the area of the shaded sector, to the nearest tenth.





4. The minute hand on a clock is 10 centimeters long and travels through an arc of 108° every 18 minutes. Find the measure of the length of the arc the minute hand travels through during this 18-minute period.