Great OR Less Than AND

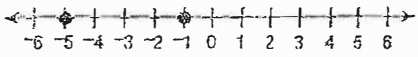
Name:_____ Date: _____

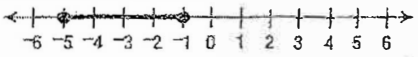
$x+3 \geq 2$ or $x+3 \leq -2$

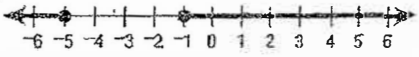
$x \geq -1$ or $x \leq -5$

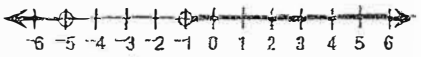
PRACTICE 1:

1. Which graph represents the solution set for $|x + 3| \geq 2$?

A. 

B. 

C. 

D. 

2. What is the solution set for $|3x| + 2 < 14$?

A. $x < -4$ $|3x| < 12$

B. $x > 4$ $3x < 12$ and $3x > -12$

C. $x < -4$ or $x > 4$ $x < 4$ and $x > -4$

D. $-4 < x < 4$

3. What is the solution set for $|2x - 7| \leq 3$?

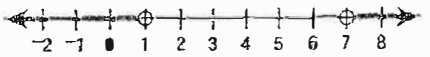
A. $2 \leq x \leq 5$ $2x - 7 \leq 3$ and $2x - 7 \geq -3$

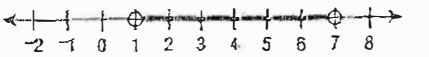
B. $-5 \leq x \leq 2$ $2x \leq 10$ and $2x \geq 4$

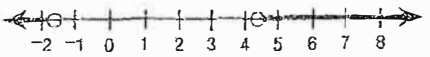
C. $x \leq -5$ or $x \geq 2$ $x \leq 5$ and $x \geq 2$

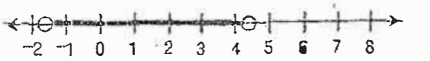
D. $x \leq 2$ or $x \geq 5$

4. Which graph best represents the solution set for $3|x - 4| < 9$?

A. 

B. 

C. 

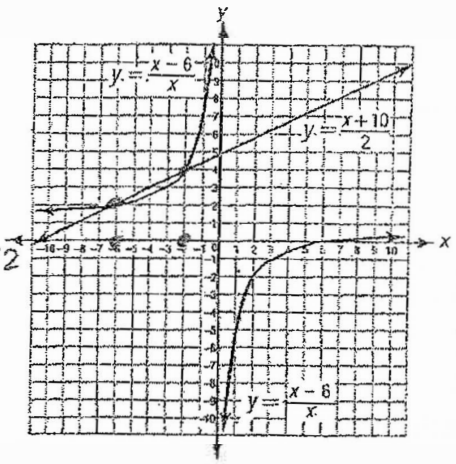
D. 

$3|x-4| < 9$

$x - 4 < 3$ and $x - 4 > -3$

$x < 7$ and $x > 1$

4. The functions $y = \frac{x+10}{2}$ and $y = \frac{x-6}{x}$ are graphed below.



Use the graph to solve $\frac{x+10}{2} = \frac{x-6}{x}$ for x.

A. $x = 6$ only

B. $x = -6$ or $x = 2$

C. $x = -2$ or $x = 4$

D. $x = -2$ or $x = -6$

5. Solve the equation for x:

$$\frac{x}{16-x} = \frac{2}{x+2}$$

A. $x = -8$ or $x = 4$

B. $x = -4$ or $x = 8$

C. $x = -2$ or $x = 8$

D. $x = 2$ or $x = 8$

You can cross multiply when you have 2 fractions joined by an equal sign.

$x(x+2) = 2(16-x)$

$x^2 + 2x = 32 - 2x$

$x^2 + 4x - 32 = 0$

$(x+8)(x-4) = 0$

$x+8 = 0$, $x - 4 = 0$

$x = -8$ $x = 4$

 Name:_____ Date:_____

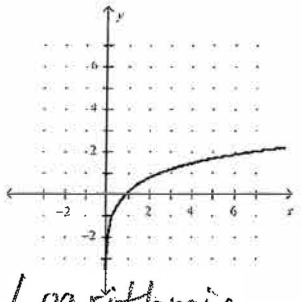
SOL Review Topic 5: Graphs of Functions

Logs, Exponentials, Absolute Value, Quadratics, Higher Order Polynomials, Cube, Cube Root, Square Roots, Rational Equations

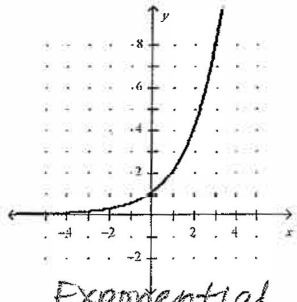
(Increasing, Decreasing, Domain, Range, Transformations, Asymptotes, Inequalities)

Recognizing Graphs of Functions

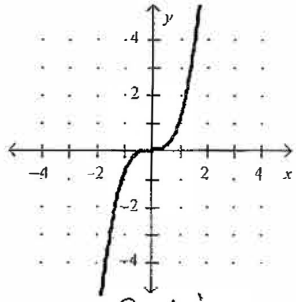
What is the name of the function show in each graph below? What is the equation of the graph?

1) 

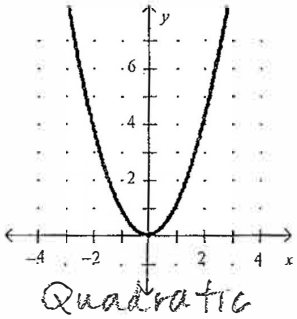
Logarithmic
$y = \log x$

2) 

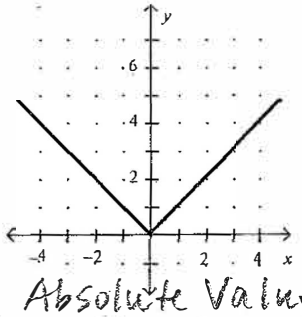
Exponential
$y = 2^x$

3) 

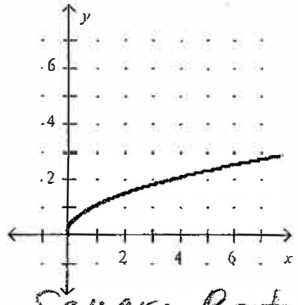
Cubic
$y = x^3$

4) 

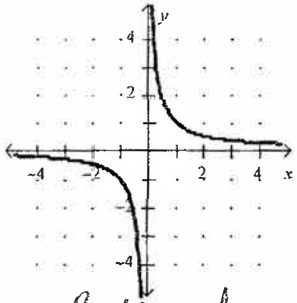
Quadratic
$y = x^2$

5) 

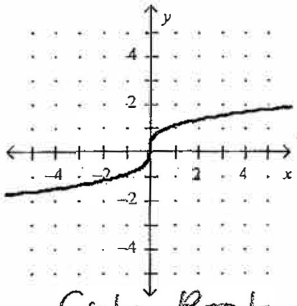
Absolute Value
$y = |x|$

6) 

Square Root
$y = \sqrt{x}$

7) 

Rational
$y = \frac{1}{x}$

8) 

Cube Root
$y = \sqrt[3]{x}$

8) Which of the above graphs have a domain or all real numbers?

Exponential (2), Cubic (3), Quadric (4), Abs Value (5), Cube Root (8)

9) Which of the above functions have a range of all real numbers?

Logrithmic (1), Cubic (3), Cube Root (8)

10) Which of the above functions have asymptotes? What are the equations of the asymptotes?

Logarithmic
$y = \log x$
Asy: $x = 0$

Exponential
$y = 2^x$
Asy: $y = 0$

Rational
$y = \frac{1}{x}$
Asy: $x = 0$ and $y = 0$

Transformation Equations

For each of the following, name the function and the vertex (or pivot point). Then give the equation of the function after it has been shifted right by three and down 2.

11) $y = 2(x-3)^2 - 5$

Name: Quadratic

Vertex: (3, −5)

Translated Equation:

$y =$ (6, −7)

12) $y = (x-1)^3$

Name: Cubic

Vertex: (1, 0)

Translated Equation:

$y =$ (4, −2)

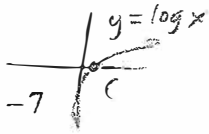
13) $y = |x-6|$

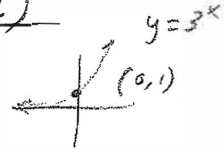
Name: Abs Value

Vertex: (6, 0)

Translated Equation:

$y =$ (9, −2)





14) $y = \sqrt{x+4} + 15$

Name: Square Root

Vertex: (−4, 15)

Translated Equation:

$y =$ (−1, 13)

15) $y = \log(x+1) - 7$

Name: Log

Vertex: (0, −7)

Translated Equation:

$y =$ (3, −9)

16) $y = 3^x - 1$

Name: Exponential

Vertex: (0, 0)

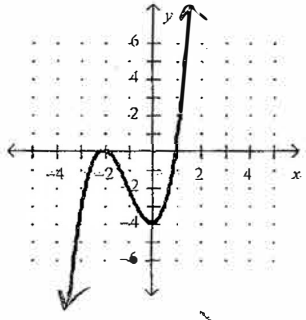
Translated Equation:

$y =$ (3, −2)

Domain, Range, Increasing Decreasing

For each of the following, determine the domain, range, intervals to which the function is increasing and decreasing, is sign of the leading coefficient and the end behavior.

17)



Domain: (−∞, ∞)

Range: (−∞, ∞)

Increasing: (−∞, −2) ∪ (0, ∞)

Decreasing: (−2, 0)

As $x \to \infty$, $f(x) \to$ +∞

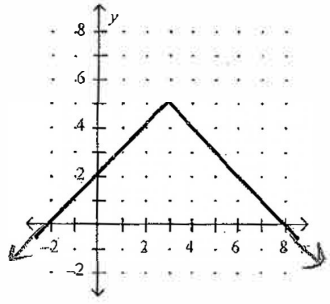
As $x \to -\infty$, $f(x) \to$ −∞

Leading Coefficient: Positive

Factors: (x+2), (x−1)

Possible Equation: $f(x) = (x-1)(x+2)^2$

18)



Domain: (−∞, ∞)

Range: (−∞, 5]

Increasing: (−∞, 3)

Decreasing: (3, ∞)

As $x \to \infty$, $f(x) \to$ −∞

As $x \to -\infty$, $f(x) \to$ −∞

Leading Coefficient: negative

Asymptotes

Find all asymptotes of the following functions.

19) $y = \log(x-5)$

$x = 5$

Vertical: $x = 5$

Horizontal: None

Big / Small = None

Same / Same = Coeff

Small / Big = $y = 0$

20) $y = 4^x - 1$

$y = 1$

V: none

H: $y = 1$

21) $y = \dfrac{x}{4x+1}$

$4x + 1 = 0$

$4x = -1$

$x = -\frac{1}{4}$

$y = \frac{1}{4}$

V: $x = -\frac{1}{4}$

H: $y = \frac{1}{4}$

22) $y = \dfrac{1}{3x^2 + 3x - 18}$

$0 = (3x-6)(x+3)$

$3x - 6 = 0$ $\quad x + 3 = 0$

$3x = 6$ $\quad x = -3$

$x = 2$

V: $x = 2, x = -3$

H: $y = 0$

23) $y = \dfrac{x+4}{2x-6}$

$2x - 6 = 0$

$2x = 6$

$x = 3$

V: $x = 3$

H: $y = \frac{1}{2}$

24) $y = \dfrac{4}{x} + 3$

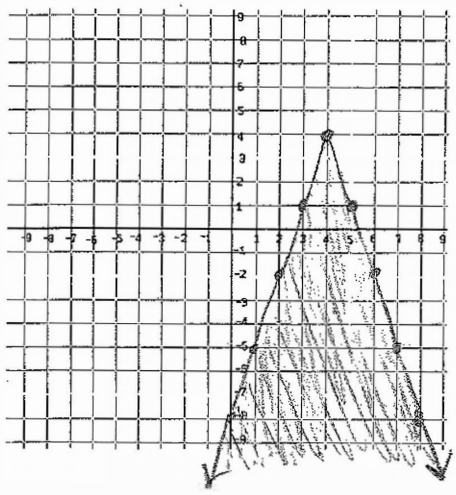
V: $x = 0$

H: $y = 0 + 3 \Rightarrow y = 3$

Inequalities

Graph the following inequality:

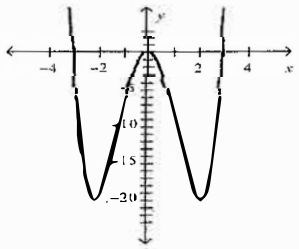
25) $y \le -3|x-4| + 4$



Zeros: Find $f(0)$ for the following functions. Name the # of real and imaginary solutions & degree.

Remember $f(0)$'s = x-intercepts = zeros = solutions = roots.

26)



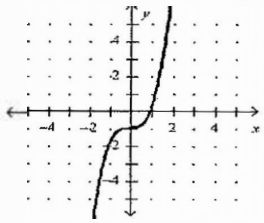
$f(0) =$ $-3, 0, 3$

# of Real Solutions = 3 (1 Double Root)

# of Imaginary Solutions = 0

Degree of Function: 4

27)



$f(0) =$ 1

# of Real Solutions = 1

# of Imaginary Solutions = 2

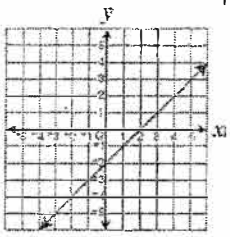
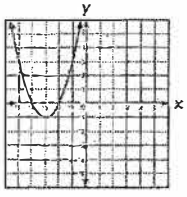
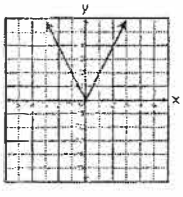
Degree of Function: 3

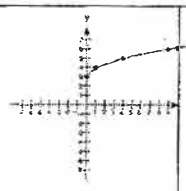
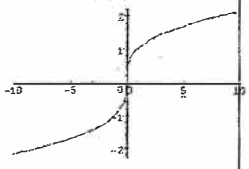
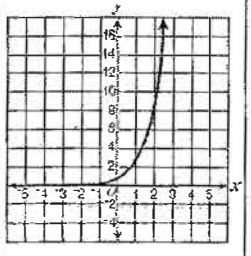
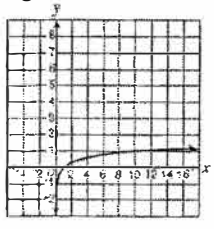
EXTRA NOTES AND EXAMPLES:

Functions

Be able to recognize the graphs for the following functions: linear, quadratic, absolute value, polynomial (cube and cube root especially), exponential, and logarithm functions.

Equation examples: $y = 2x - 3$ →Linear (degree of 1), $y = x^2$ → Quadratic (degree of 2), $y = |x|$ →Absolute Value, $y = x^3$ →Cube function, $y = \sqrt[3]{x}$ →Cube Root, $y = 2^x$ →Exponential (a number raised to the *x* power), $y = \log_2 x$ →Logarithm

Function	Equation etc
Linear 	y = mx + b (SI) ax + by = c (SF) $m = \frac{y_2 - y_1}{x_2 - x_1}$ slope b is (0,b) y-intercept (x_1, y_1) point on the line Horizontal line HOY y = #, zero slope Vertical lines are not functions (VUX) x = # undefined slope
Quadratic "U" Parabola Find Vertex by using "Calc" key maximum or minimum 	$y = a(x-h)^2 + k$ Vertex (h,k) opposite same a>0 opens up, a< 0 opens down $\|a\|$ >1 stretch, $\|a\|$ <1 shrink
Absolute Value "V" 	$y = a\|x-h\| + k$ Vertex (h,k) opposite same a>0 opens up, a< 0 opens down $\|a\|$ >1 stretch, $\|a\|$ <1 shrink
Square Root	$y = a\sqrt{x-h} + k$

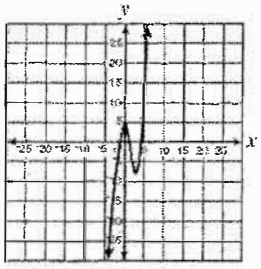
	opposite same Starting point (h, k) a>0 opens up, a< 0 reflects down $\|a\|$ >1 stretch, $\|a\|$ <1 shrink
Cube Root 	$y = a\sqrt[3]{x-h} + k$ opposite same Turning Point (h,k) a>0 as graph on left , a< 0 reflects $\|a\|$ >1 stretch, $\|a\|$ <1 shrink
Exponential Growth 	$y = a(b)^{(x-h)} + k$ b > 1 y = k is the horizontal asymptote e ≈ 2.72 (natural log base e)
Logarithmic 	$y = \log_b(x-h) + k$ (inverse of exponential). $\log_b(a) = \frac{\log a}{\log b}$ x = h is vertical asympt. Log is log base 10 Ln is log base e

EXTRA NOTES AND EXAMPLES:

Polynomials: To find the zeros of a polynomial equation, either:

1.) Graph the equation on your calculator and look at where the graph crosses/touches the x-axis

Or 2.) Solve the equation by factoring and setting each factor = 0 (may need to use the quadratic formula (given to you on the 'formula' screen.) You must do this when you cannot tell where the graph crosses or if it doesn't cross the *x*-axis.)

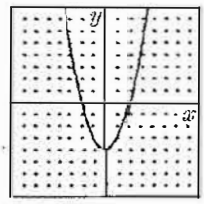
Polynomials Example: Cubic Degree 3 	Zeros	Types	Turns	End Behavior
	1. Real Zeros are the x values of the x intercepts. 2. Zeros are also called *roots*, or *solutions* 3. If the zero is $x = h$, then its factor is (x-h) 4. The number of zeros = the degree (this includes real, imaginary and double roots)	1. If there are no x ntercepts there are no real zeros, (all zeros will be imaginary) 2. A tangent implies a double root (repeated solution) 3. Irrational zeros come in pairs as do imaginary zeros	1. The maximum number of turns is equal to the degree – 1.	1. If the leading coefficient (LC) is '+' the right behavior rises, if the LC is '-' the right behavior falls 2. If the degree is *even*, right and left behavior will be the same, if the degree is *odd* right and left behavior is opposite.

Finding Domain/Range,

A 'Function' means that *x*-values do not repeat---it must pass the vertical line test.

Domain – set of all *x*-values Range – set of all *y*-values

Ex 1: Find the Domain/Range of $y = x^2 - 3$.

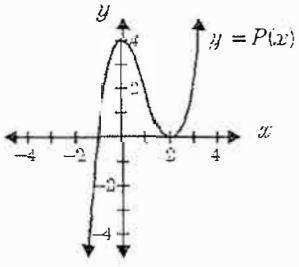


From the graph shown: (Note: $\Re$ symbol means "all reals")

Domain = All Real Numbers

Range = All Numbers Greater than -3

Increasing/Decreasing Intervals



As *x* increases from - infinity to + infinity (read from left → right), do *y* values increase or decrease? The intervals will be the *x* values in these areas.

Ex 3: What are the decreasing intervals? The function decreases from (0, 2)

EXTRA NOTES AND EXAMPLES:

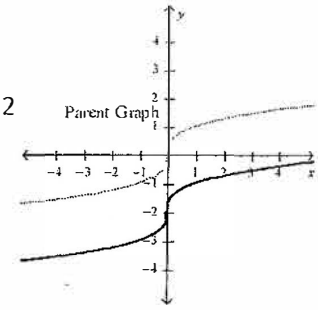
Leading Coefficients

If the function ends up, the leading coefficient is positive. If the function ends down, the leading coefficient is negative.

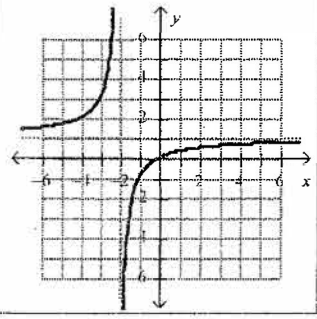
Transformations

What is the new equation shown in bold in the graph to the right?

The parent graph is the cube root function $y = \sqrt[3]{x}$. The function is shifted down by 2 therefore the new equation is $y = \sqrt[3]{x} - 2$



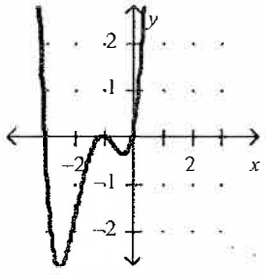
Rational Functions: See the chart for information on rational graphs:

Rational Function			
	$y = \frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomial functions $q(x) \neq 0$ discontinuous	Domain all real numbers except the values that make q(x)= 0 Zeros of function set p(x)= 0 and solve	Vertical Asymptotes: Set q(x) = 0 and solve. Look at domain restrictions. Horizontal Asymptotes: 1. Degree of p(x) < Degree of q(x) y = 0 2. Degree of p(x) > Degree of q(x) None 3. Degree of p(x) = Degree of q(x) y = LC of p(x)/LC of Q(x)

Factors, Zeros and Equations

What is the sign of leading coefficient of the graph to the right?

The leading coefficient is positive because the function ends up.



Determine the end behavior.

As $x \to +\infty, f(x) \to +\infty$ and As $x \to -\infty, f(x) \to +\infty$

What are the factors?

$x, (x+3), (x+1)$

What is a possible equation?

$f(x) = x(x+3)(x+1)^2$

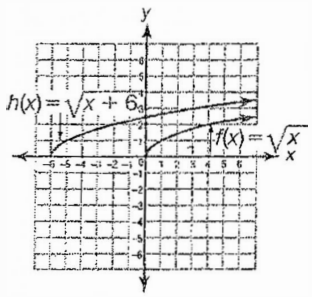
What are the zeros of the function? Remember - $f(0)$'s = x-intercepts = zeros = solutions = roots.

$\{-3, -1, 0\}$

J _____ Date: _____

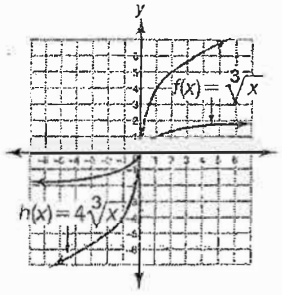
PRACTICE J:

1. Which describes how the graph of $f(x) = \sqrt{x}$ could be transformed to form the graph of $h(x) = \sqrt{x + 6}$?



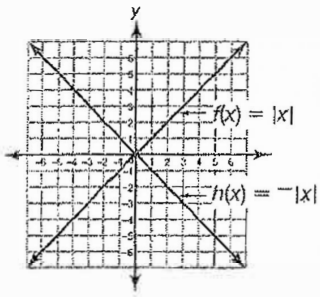
A. translation of 6 units up
B. translation of 6 units to the left
C. translation of 6 units to the right
D. dilation (vertical shrink)

2. Which describes how the graph of $f(x) = \sqrt[3]{x}$ could be transformed to form the graph of $h(x) = 4\sqrt[3]{x}$?



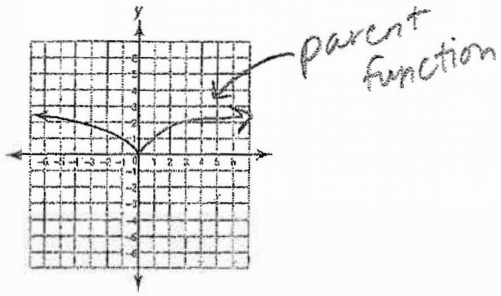
A. reflection across the y-axis
B. translation 3 units to the left
C. translation 3 units down
D. dilation (vertical stretch)

3. Which describes how the graph of $f(x) = |x|$ could be transformed to form the graph of $h(x) = -|x|$?



A. translation of 1 unit down
B. translation of 1 unit up
C. reflection across the x-axis
D. reflection across the y-axis

4. The graph below was created by reflecting the graph of its parent function over the y-axis.

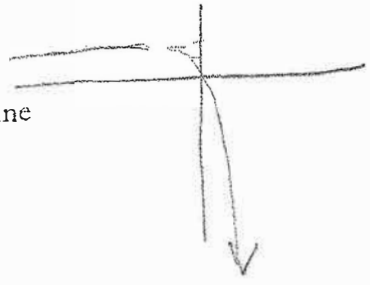


Which equation names its parent function?

A. $f(x) = \sqrt{x}$
B. $f(x) = \sqrt[3]{x}$
C. $f(x) = |x|$
D. $f(x) = \frac{1}{x}$

7. Which statement is **not** true of the function $f(x) = -2(4^x) + 2$?

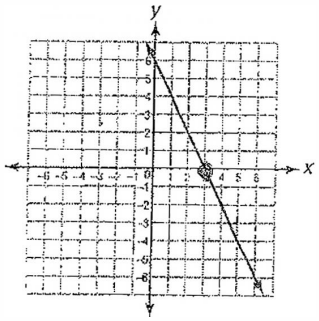
Graph it!



A. It is increasing.
B. Its x-intercept is the same as its y-intercept, $(0, 0)$.
C. Its horizontal asymptote is the line $y = 2$.
D. Its range is $y < 2$.

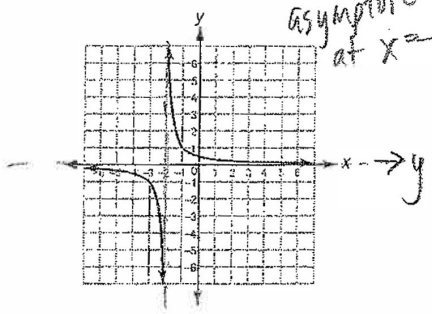
PRACTICE K:

1. Below is the graph of $f(x) = -2x + 6$. What is the x-intercept of this function?



A. (0, 6)
B. (0, 3)
C. (3, 0)
D. (6, 0)

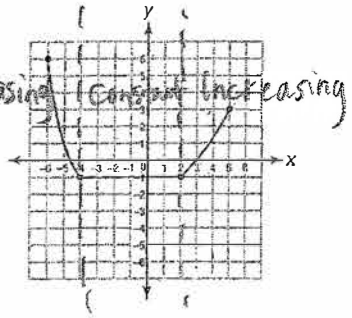
Below is the graph of $f(x) = \frac{1}{x+2}$.



6. What is the domain of this function? (Use a graphing calculator to confirm your results.)
   A. the set of all real numbers except -2
   B. the set of all real numbers except 0
   C. the set of all real numbers except 2
   D. the set of all real numbers because the function is continuous

7. What are the asymptotes for this function?
   A. $x = 0$ and $y = 0$
   B. $x = 0$ and $y = -2$
   C. $x = -2$ and $y = -2$
   D. $x = -2$ and $y = 0$

Below is the graph of a function.



8. On what interval is the function increasing?
   A. $(-\infty, -6)$
   B. $(-4, 5)$
   C. $(-4, 2)$
   D. $(2, 5)$

9. Which statement is **not** true about the function graphed above?
   A. It is decreasing on the interval $(-6, -4)$.
   B. It is constant on the interval $(-\infty, -6)$.
   C. It has a y-intercept at $(0, -1)$.
   D. Its domain is: $-6 \leq x \leq 5$.

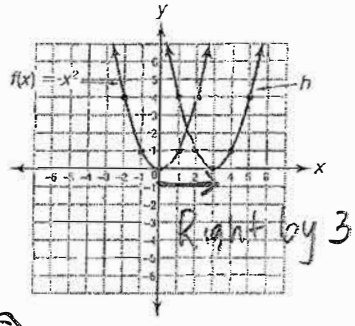
Horizontal Asymptotes

$\frac{\text{BIG}}{\text{Small}}$ = None $\frac{\text{Small}}{\text{Big}}$ = $y = 0$

$\frac{\text{Same}}{\text{Same}}$ = Ratio of Coefficients of Highest Degree Variable

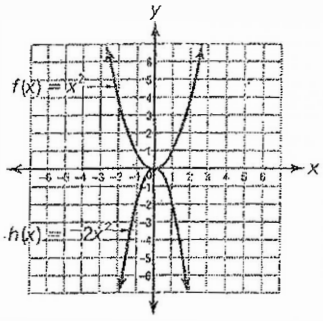
PRACTICE L:

3. The parent graph $f(x) = x^2$ was transformed to form the graph of the function h shown below. What is the equation of the resulting graph?



A. $h(x) = (x - 3)^2$
B. $h(x) = (x + 3)^2$
C. $h(x) = x^2 - 3$
D. $h(x) = x^2 + 3$

4. Which could describe how the graph of $f(x) = x^2$ could be transformed to form the graph of $h(x) = -2x^2$ in two steps?



A. reflection across the y-axis followed by a vertical shift of 2 units up
B. reflection across the x-axis followed by a vertical shift of 2 units up
C. reflection across the y-axis followed by a vertical stretch by a factor of 2
D. reflection across the x-axis followed by a vertical stretch by a factor of 2

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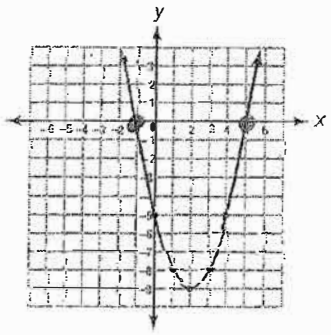
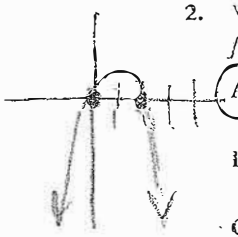
1. The graph below shows $f(x) = x^2 - 4x - 5$. Which statement is true of this function?



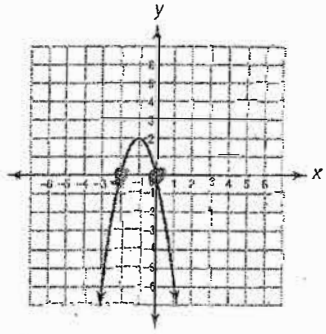
A. Its domain is $2 \le x \le 6$.
B. Its range is $y \le -9$.
C. Its zeros are -1 and 5.
D. Its y-intercept is (0, 5).

2. Which is true of the graph of $f(x) = -x^2 + 2x$?



A. It is increasing on the interval $\{-\infty \le x < 1\}$.
B. It is decreasing on the interval $\{-\infty \le x < 1\}$.
C. It has a y-intercept at (0, 2).
D. It has only one x-intercept, (0, 0).

Below is the graph of $f(x) = -2x^2 - 4x$.



3. What is the range for this function?

A. the set of all real numbers
B. $y \le 2$
C. $y \ge 2$
D. $-2 \le y \le 0$

4. Which is not true of this function?

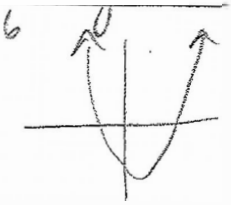
A. Its zeros are -2 and 0.
B. $(-2, 0)$ is an x-intercept for this function.
C. (0, 0) is an x-intercept and a y-intercept for this function.
D. It is increasing on the interval $\{-\infty < x < 1\}$.

Name:_____ Date: _____

PRACTICE M:

$-2y = -2x^2 + 6x + 26$

$y = x^2 - 3x - 13$ Graph it!



7. What is the end behavior for the graph of the function $2x^2 - 2y = 6x + 26$?

   A. Both arms of the parabola point down.

   B. Both arms of the parabola point up.

   C. The left arm points down, and the right arm points up.

   D. The left arm points up, and the right arm points down.

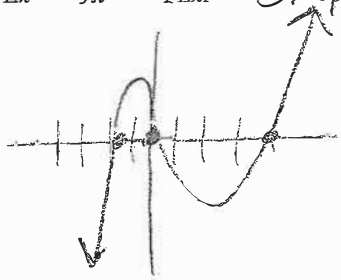
2. Which of the following is **not** a zero for the function $f(x) = 2x^3 - 5x^2 - 12x$? Graph it!

   A. -4 ✗

   B. $-\frac{3}{2}$ ✓

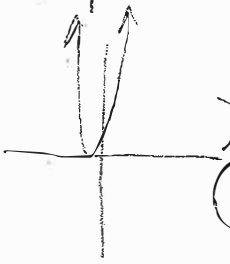
   C. 0 ✓

   D. 4 ✓



Graph it.

8. What are the intercepts of the function $f(x) = 16x^2 + 8x + 1$? $y = (4x+1)^2$



   A. x-intercept: $\left(-\frac{1}{4}, 0\right)$, y-intercept: $(0, 0)$ ✓

   B. x-intercept: $\left(\frac{1}{4}, 0\right)$, y-intercept: $(0, 1)$ ✗

   C. x-intercepts: $\left(-\frac{1}{4}, 0\right)$ and $\left(\frac{1}{4}, 0\right)$, y-intercept: $(0, 1)$ ✓ $4x+1=0$, $x = -\frac{1}{4}$

   D. x-intercepts: $\left(\frac{1}{4}, 0\right)$ and $(4, 0)$, y-intercept: $(0, 1)$ ✗

$y = 16(0)^2 + 8(0) + 1$

$y = 1$.

3. Which describes the end behavior for the function $f(x) = x^5 + 2x^3 - 8x + 1$?

   A. Both arms point up.

   B. Both arms point down.

   C. The left arm points up, and the right arm points down.

   D. The left arm points down, and the right arm points up.

5th degree so one end ↑ one end ↓

positive leading coefficient so must end ↑ therefore start ↓

Graph it to check.

Use a graphing calculator for question 7 and 8.

7. What is the range of the function $f(x) = x^6 - 3$?

   A. the set of all real numbers

   B. $y \leq 0$

   C. $y \leq -3$

   D. $y \geq -3$ $[-3, \infty)$

8. Which is true of the function $f(x) = -2x^4 + 1$?

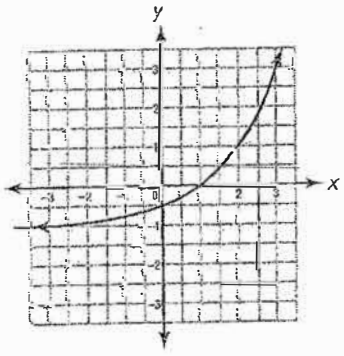
   A. It is increasing on the interval $\{-\infty < x < 0\}$. $(-\infty, 0)$

   B. It is increasing on the interval $\{0 < x < \infty\}$. ✗

   C. It has a y-intercept at $(0, 0)$. ✗

   D. It has four real zeros. ✗

Use the graph of $f(x) = 2^{x-1} - 1$ for questions 5 and 6.



5. What is the asymptote of this function?

   A. the line $x = 0$

   B. the line $y = -1$

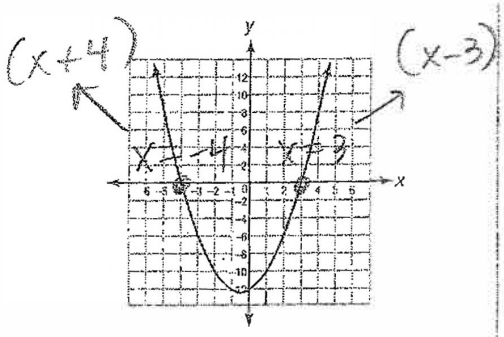
   C. the line $y = 0$

   D. the line $y = 1$

PRACTICE N:

You may use a graphing calculator for questions 1–8.

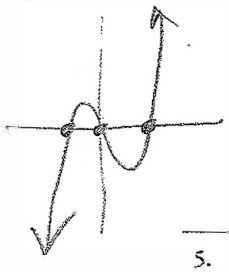
1. The graph of $f(x) = x^2 + x - 12$ is shown below.



Use this graph to determine the factored form of $x^2 + x - 12$.

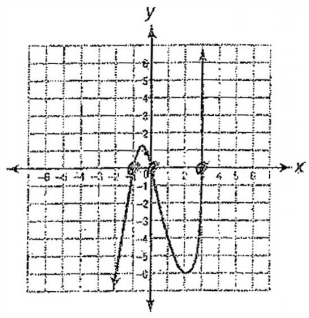
A. $(x - 4)(x - 3)$
B. $(x - 3)(x + 4)$
C. $(x - 4)(x + 3)$
D. $(x - 0)(x - 12)$

2. Which statement about the roots of $x^3 - 4x = 0$ is true? Graph it!



A. It has 4 real solutions.
B. It has 3 real solutions.
C. It has 2 real solutions and 1 nonreal solution.
D. It has 1 real solution and 2 nonreal solutions.

3. The graph of $f(x) = x^3 - 2x^2 - 3x$ is shown below.



x=-1 x=0 x=3
(x+1) x (x-3)

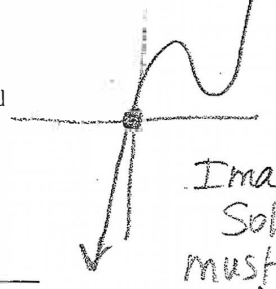
Use this graph to determine the factored form of $x^3 - 2x^2 - 3x$.

A. $x(x - 3)(x + 1)$
B. $x(x - 1)(x + 3)$
C. $x(x - 3)(x - 1)$
D. $(x - 1)^2(x + 3)$

4. Which statement about the solutions of $x^3 - 8x^2 + 17x = 0$ is true?

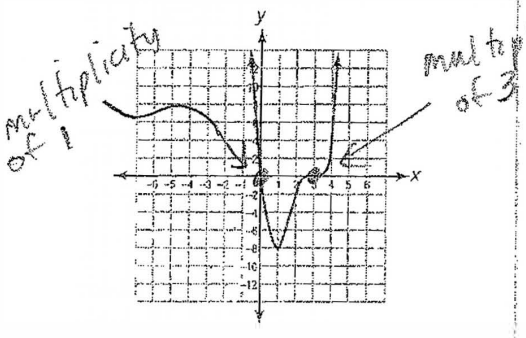
A. It has 4 real solutions.
B. It has 3 real solutions.
C. It has 2 real solutions and 1 nonreal solution.
D. It has 1 real solution and 2 nonreal solutions.

Graph it!



Imaginary Solutions must come in pairs
3rd degree therefore should be 3 solutions total

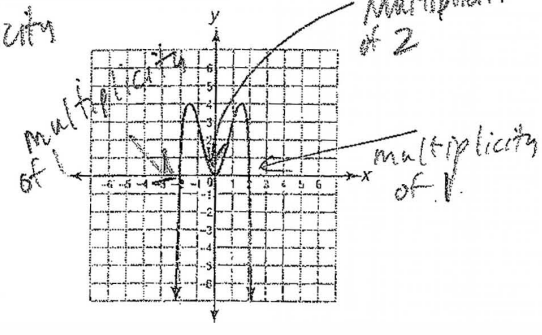
5. The graph of $f(x) = x^4 - 9x^3 + 27x^2 - 27x$ is shown below.



Which is true of the zeros of this function?

A. The zero 0 has a multiplicity of 2.
B. The zero 0 has a multiplicity of 3.
C. The zero 3 has a multiplicity of 3.
D. The zero 3 has a multiplicity of 4.

7. The graph of $f(x) = -x^4 + 4x^2$ is shown below.



Which is true of the zeros of this function?

A. The zero -2 has a multiplicity of 2.
B. The zero 0 has a multiplicity of 2.
C. The zero 2 has a multiplicity of 2.
D. There are no zeros of multiplicity.

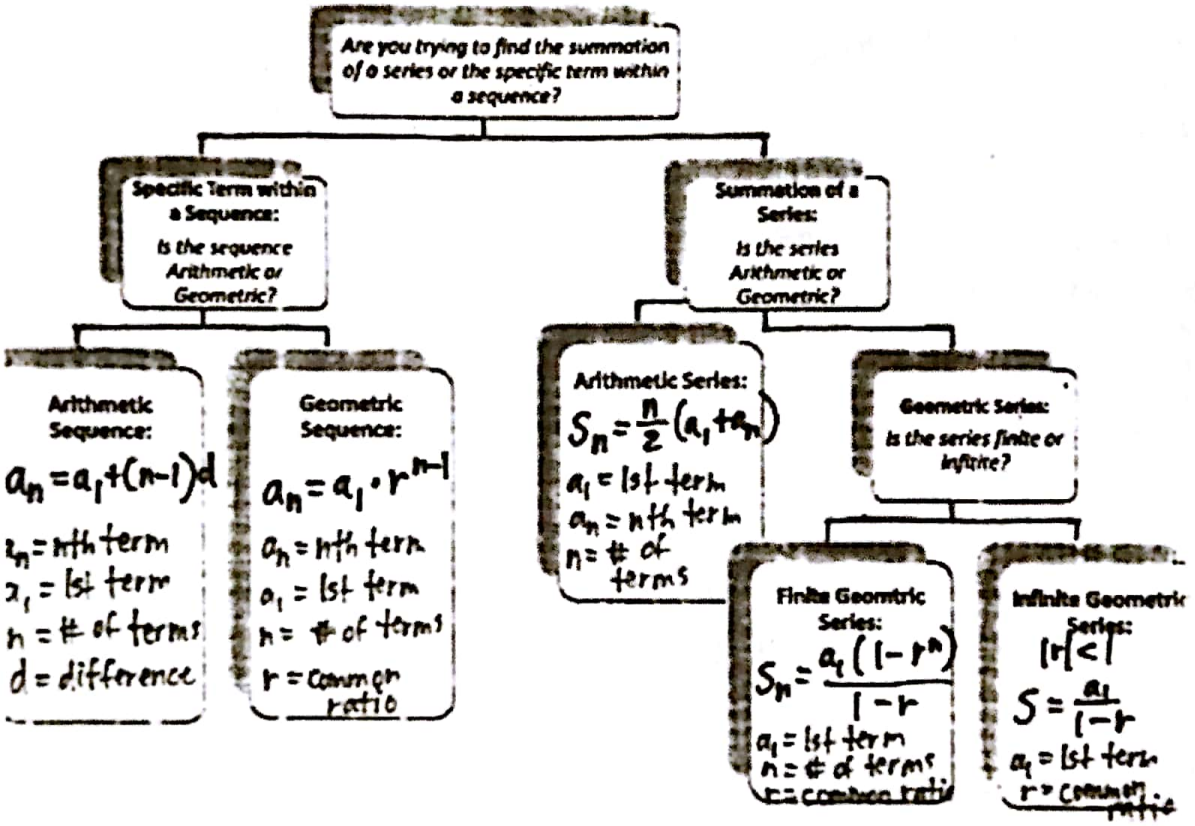
Algebra 2 Review Topic 6: Other!

Sequences and Series, Statistics, Composition of Functions, ~~Variation~~, Inverses, Properties

Sequences and Series

How do I know when to use each formula?

How do you know which formula to use for sequence and series problems?



Mixed Sequences and Series Practice

1) Find 1st 3 terms: $a_1 = 4,\ a_{n+1} = 2a_n + 1$ *for* $n \geq 1$

a_n = current term
a_{n+1} = next term
a_{n-1} = previous term

next term = 2 · current term + 1

4, 9, 19

2) Find a_{20} for $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

geometric

$r = \frac{a_2}{a_1} = \frac{1}{2}$

$a_n = a_1 \cdot r^{n-1}$

$a_{20} = 2\left(\frac{1}{2}\right)^{20-1}$

$a_{20} = \frac{1}{2^{18}}$

3) Find the 3 arithmetic means: 5, $\hat{}$, $\hat{}$, $\hat{}$, -3

3, 1, -1

$-3 - 5 = -8$

$-8 \div 4 = -2$

4) Find the 17th term if $a_1 = -20$ & $d = 4$

arithmetic

$a_n = a_1 + (n-1)d$

$a_{17} = -20 + (17-1)(4)$

$a_{17} = 44$

Mixed Sequences and Series Practice - Continued

geometric $r = \frac{a_2}{a_1} = \frac{-2}{1} = -2$

5) 97 is the ___?___ th term of $-3, 1, 5, 9, \ldots$

arithmetic
$d = a_2 - a_1$
$d = 1-(-3) = 4$

$97 = -3 + (n-1)4$
$97+3 = (n-1)4$
$100 = 4(n-1)$
$25 = n-1$
$n = 26$

6) Find sum of $1-2+4-8+16\ldots$ to 15 terms $n=15$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_{15} = \frac{1(1-(-2)^{15})}{1-(-2)} = 10923$

7) Find the sum of geometric series $a_1 = 10,\ a_n = 270,\ n = 4$

need r
$a_n = a_1 r^{n-1}$
$270 = 10 \cdot r^{4-1}$
$27 = r^3$
$r = \sqrt[3]{27} = 3$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_4 = \frac{10(1-3^4)}{1-3}$

$S_4 = 400$

8) Find the sum of $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$ $r = \frac{a_2}{a_1} = \frac{1}{2}$

Infinite sum: $|r| < 1$

$S_\infty = \frac{a_1}{1-r} = \frac{2}{1-\frac{1}{2}} = 4$

Statistics 1 – Finding Regression Equation

9) Jean invested $380 in stocks. Over the next 5 years, the value of her investment grew, as shown in the accompanying table. Write the regression equation for this set of data, rounding all values to *two decimal places*. Using this equation, find the value of her stock, to the *nearest dollar*, 10 years after her initial purchase.

Years Since Investment (x)	Value of Stock, in Dollars (y)
0	380
1	395
2	411
3	427
4	445
5	462

Stat Edit
Turn Stat Plot on
Zoom 9

Linear regression
Stat → Calc → LinReg(ax+b)

$y = 16.46x + 378.86$

$y = 16.46(10) + 378.86 = 543.46 \approx \$543$

Statistics 2 – Fundamental Counting Rule, Permutations, Combinations

10) In the next Olympics, the United States can enter four athletes in the diving competition. How many different teams of four divers can be selected from a group of nine divers?

$9C_4 = 126$

11) Find the total number of different twelve-letter arrangements that can be formed using the letters in the word PENNSYLVANIA.

$12P_{12} = 479{,}001{,}600$

12) A four-digit serial number is to be created from the digits 0 through 9. How many of these serial numbers can be created if 0 can not be the first digit, no digit may be repeated, and the last digit must be 5?

$8 \cdot 8 \cdot 7 \cdot 1 = 448$

not 0 or 5

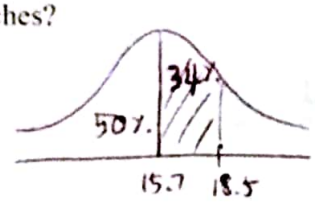
13) A multiple choice test has 10 questions where each question has 4answers. If you select one of the four answers for each question, how many different ways can you answer the questions?

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

$= 4^{10} = 1048576$

Statistics 3 – Normal Distribution and Z-Scores

14) The width of shark jaws are normally distributed with a mean of 15.7 and a standard deviation of 2.8 inches. What is the probability that a shark that you examine at random has a jaw width less than 18.5 inches?

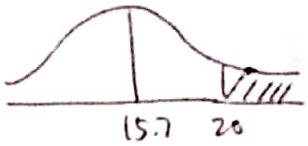


$z = \frac{18.5 - 15.7}{2.8} = 1$

50% + 34% = 84%.

P(shark width < 18.5) = 0.84

15) What is the probability that a shark that you examine at random has a jaw greater than 20 inches?



$z = \frac{20 - 15.7}{2.8} = 1.54$ $\left(z = \frac{x - \mu}{\sigma}\right)$

use table!

$P(z > 1.54) = 1 - P(z < 1.54)$

$= 1 - .9382 = 0.0618$ (from table)

approx. 6%.

Composition of Functions

16) If $f(x) = \sqrt{x} + 1$ and $g(x) = x + 3$, then find $f \circ g$.

$f(g(x)) = f(x+3) = \sqrt{x+3} + 1$

17) If $f(x) = \frac{1}{x}$ and $g(x) = x^2 - x$, find $f(g(-1))$.

$f(g(-1)) = f(2) = \frac{1}{2}$

$g(-1) = (-1)^2 - (-1) = 2$

Inverses

1 to 1 function

18) Find the inverse of $y = \frac{1}{2}x - 2$.

Switch x and y: $x = \frac{1}{2}y - 2$

$x + 2 = \frac{1}{2}y$

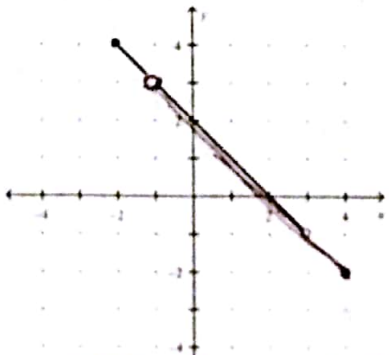
$2x + 4 = y$

$f^{-1}(x) = 2x + 4$

19) Is $y = x^2 - 2$ a one-to-one function?

Not a 1-to-1 function. Does not pass horizontal line test so its inverse is not a function.

20) Graph the inverse of the line segment.



original vs. inverse

(-2, 4) (4, -2)

(3, -1) (-1, 3)

swap x, y

21) What is the range of the graphed line segment?

(-1, 4]

22) What is the domain of the inverse? the same

(-1, 4]

23) Graph $y = 10^x$ and the inverse of $y = 10^x$.

Inverse: $x = 10^y$

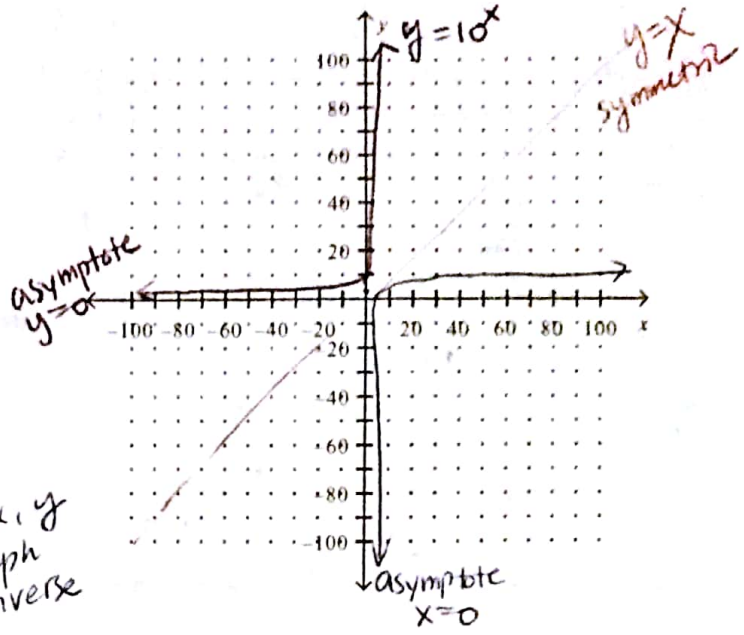
$y = \log_{10} x$

$f^{-1}(x) = \log_{10} x$

$y = 10^x$

x	y
0	1
1	10
2	100
-1	$\frac{1}{10}$
-2	$\frac{1}{100}$

Switch x, y and graph inverse



Polynomial / Synthetic division

24) Divide: $\dfrac{-13x^2 + 4x^3 + 2x - 7}{x^2 + 3x - 2}$

$4x - 25$

$x^2+3x-2 \;\overline{)\; 4x^3 - 13x^2 + 2x - 7}$

$-(4x^3 + 12x^2 - 8x)$

$-25x^2 + 10x - 7$

$-(-25x^2 - 75x + 50)$

$85x - 57$

$4x - 25 + \dfrac{85x - 57}{x^2 + 3x - 2}$

25) Divide: $\dfrac{2x^3 + 5x^2 + 9}{x + 3} = 0$ ---- $x = -3$

-3	2	5	0	9
	↓	-6	3	9
	2	-1	3	0
	↑	↑	↑	
	x^2	x	#	

$2x^2 - x + 3$

Log/exponential equations base is always base

Convert each log expression into an exponential expression.

26) $\log_{12} 144 = 2$

$144 = 12^2$

27) $\log_4 \frac{1}{64} = -3$

$\frac{1}{64} = 4^{-3}$

28) $\log_{27} 3 = \frac{1}{3}$

$3 = 27^{\frac{1}{3}}$

Convert each exponential expression into a log expression.

29) $6^2 = 36$

$2 = \log_6 36$

30) $2^{-5} = \frac{1}{32}$

$-5 = \log_2 \frac{1}{32}$

31) $m^n = p$

$n = \log_m p$

$y = 3x + 2$

$x = 3y + 2$

$x - 2 = 3y$

$\frac{x-2}{3} = y$ ✓

PRACTICE

8. Which pair of functions are inverses? Use composition to determine the answer.

   A. $f(x) = 3x + 2$ and $g(x) = 3x - 2$

   B. $f(x) = 3x + 2$ and $g(x) = \frac{x-2}{3}$

   C. $f(x) = 3x$ and $g(x) = \frac{3}{x}$

   D. $f(x) = 3x$ and $g(x) = x - 3$

1. Given $f(x) = \frac{x}{3}$ and $g(x) = 6x + 9$, which is equal to $f(g(x))$?

   $f(6x+9)$

   $= \frac{6x+9}{3} = 2x+3$

   A. x

   B. $2x + 3$

   C. $2x + 9$

   D. $6x + 9$

2. If $h(x) = 2x$ and $j(x) = 3x^2$, then which is equal to $j(h(x))$?

   $j(3x^2)$

   $= 2(3x^2)$

   $= 6x^2$

   A. $6x$

   B. $6x^2$

   C. $12x$

   D. $12x^2$

3. Given $f(x) = 2x^2 - 8$ and $g(x) = x + 2$, what is the value of $f(g(5))$?

   $f(g(5))$

   $g(5) = 5 + 2 = 7$

   $f(7) = 2(7)^2 - 8$

   $= 98 - 8 = 90$

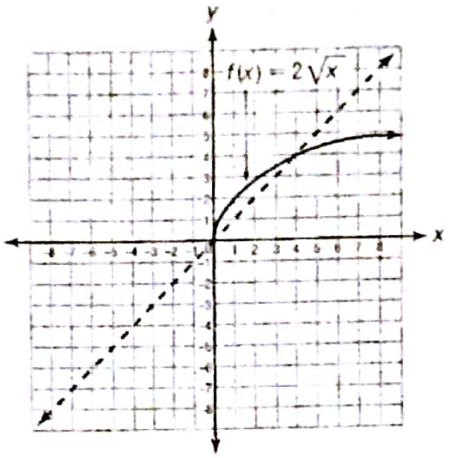
   A. 42

   B. 68

   C. 90

   D. 108

6. The graph of $f(x) = 2\sqrt{x}$ is shown on the grid below.



Which is **not** a point on the inverse of this function?

swap x, y

   A. $(0, 0)$ $(0,0)$ ✓

   B. $(2, -1)$ $(-1,2)$

   C. $(4, 4)$ $(4,4)$ ✓

   D. $(6, 9)$ $(9,6)$ ✓

7. Which of the following is the equation of the inverse of $f(x) = \frac{2}{x}$?

$y = \frac{2}{x}$

$\frac{x}{1} = \frac{2}{y}$

$2 = xy$

$y = \frac{2}{x}$

$f^{-1}(x) = \frac{2}{x}$

   A. $f^{-1} = \frac{2}{x}$

   B. $f^{-1} = \frac{x}{2}$

   C. $f^{-1} = 2x$

   D. $f^{-1} = 2$

5. There are 8 books on a shelf. If 3 books are chosen at random, how many different groups of 3 books could be chosen?

A. 19 **C.** 56

B. 28 **D.** 65

$8C_3$

6. In a figure skating competition, the order in which skaters compete is determined by a drawing. Suppose there are 6 skaters in a competition. In how many different ways can a first and second skater be chosen?

A. 720 **C.** 30

B. 360 **D.** 15

$6P_2$

7. For a civil trial, 5 people out of a panel of 11 people must be chosen to deliberate as jurors. How many different groups of 5 jurors could be selected from the panel of 11 people?

A. 120

B. 462

C. 55,440

D. 332,640

$11C_5$

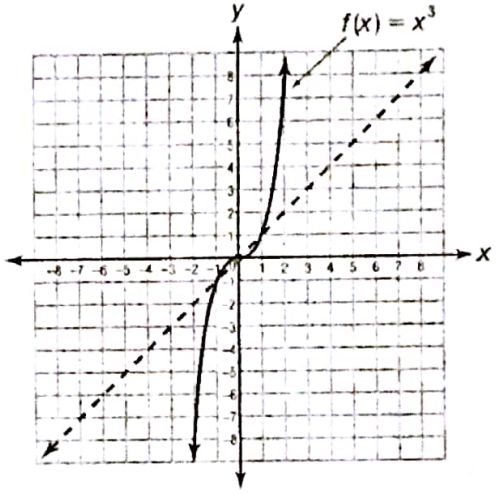
8. To log on to the school computer system, each student must choose a password with 2 digits and 2 letters, in that order. Each letter and digit may be used only once. How many different passwords are possible?

A. 90 **C.** 14,950

B. 650 **D.** 58,500

$10 \cdot 9 \cdot 26 \cdot 25$

4. The graph of $f(x) = x^3$ is shown on the grid below.



Which is **not** a point on the inverse of this function?

A. $(-8, -2)$ $(-2, -8)$

B. $(-1, 1)$ $(1, -1)$ X ---> $(1, 1)$

C. $(0, 0)$ $(0, 0)$

D. $(8, 2)$ $(2, 8)$

5. Which of the following is the inverse of $f(x) = 9^x$?

$y = 9^x$

$x = 9^y$

$y = \log_9 x$

A. $f^{-1} = \log_9 x$

B. $f^{-1} = 9 \log x$

C. $f^{-1} = \frac{x}{9}$

D. $f^{-1} = 9x$

PRACTICE Q:

CT : current term
NT : next term
PT : previous term

1. Which recursive formula describes the geometric sequence shown below?

$-6, -24, -96, -384, \ldots$ $r = \frac{a_2}{a_1} = \frac{-24}{-6} = 4$

A. $a_n = a_{n-1} \cdot 4$ CT = PT · 4
B. $a_n = a_{n-1} \cdot -4$ CT = PT · (−4)
C. $a_n = a_{n-1} \cdot -6$ CT = PT(−6)
D. $a_n = a_{n-1} \cdot -18$ CT = PT · (+18)

4. Look at the notation below. What is the indicated sum for the arithmetic series?

$\sum_{k=1}^{18} (10 - 2k)$ 18 terms

A. 306 8, 6, 4, ···, −26
B. −26
C. −162 $S_{18} = \frac{18}{2}(8 + -26)$
D. −468

$S_{18} = -162$

5. What is the indicated sum for this geometric series?

S_6 for 4, 20, 100, 500, 2500, 12500

A. 624
B. 3,124
C. 12,500
D. 15,624

or $a_1 = 4$ $r = \frac{20}{4} = 5$

$S_6 = \frac{a_1(1-r^6)}{1-r}$
$= \frac{4(1-5^6)}{1-5}$
$= 5^6 - 1$
$= 3124$

4. Look at the notation below. What is the indicated sum for the arithmetic series?

$\sum_{k=1}^{18} (10 - 2k)$

A. 306
B. −26
C. −162
D. −468

Use the information below for questions 6 and 7.

Gamal is buying a new laptop computer on layaway. He paid $25 initially and will increase his payment each week. His planned payments will form an arithmetic sequence, as shown in the table. If he follows them, his laptop will be fully paid for in 11 weeks.

Week (n)	Payment in Dollars (a_n)
1	$25
2	$35
3	$45
4	$55
...	...
11	

$a_1 =$ add 10 $a_n =$

Linear Regression

6. What will Gamal's planned payment be in Week 11?

A. $75
B. $125
C. $135
D. $275

$y = 10x + 15$
$y = 10(11) + 15$
$y = 125$

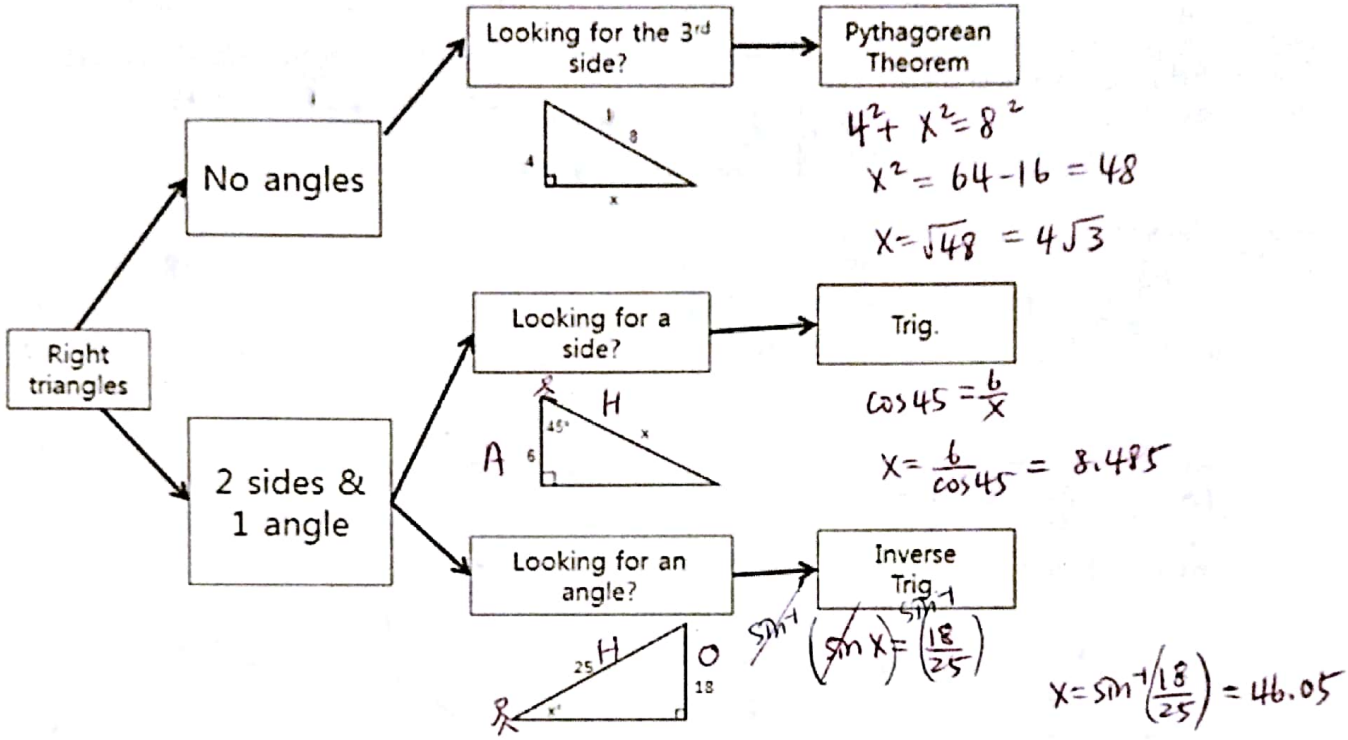
7. What will be the total amount paid for the laptop in 11 weeks?

A. $300
B. $825
C. $880
D. $1,375

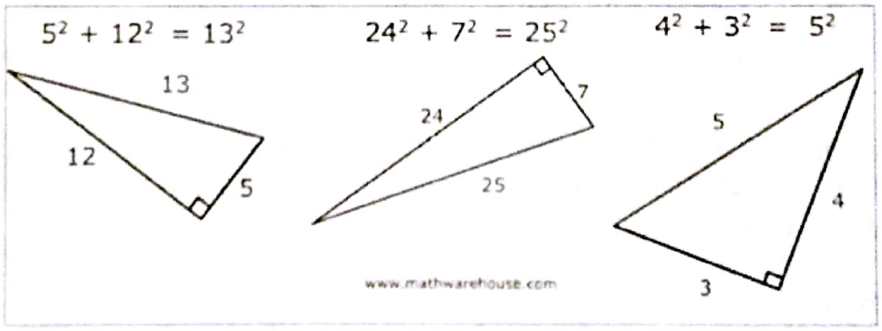
$S_n = \frac{n}{2}(a_1 + a_n)$
$S_{11} = \frac{11}{2}(25 + 125)$
$S_{11} = 825$

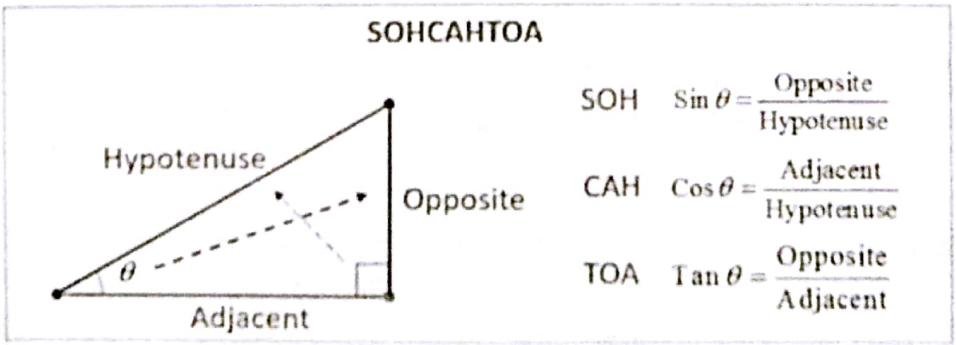
Topic 7 Trigonometry Review:

Pythagorean Theorem, SOH CAH TOA and word problems

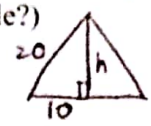


Pythagorean Theorem example: set hypotenuse (longest side or across from right angle) to "c"



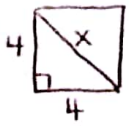


1. A side of an equilateral triangle is 20 cm long. What is the height/altitude of this triangle in the **simplest radical form**? (Do you see a right triangle?)



$h^2+10^2=20^2$
$h^2=300$
$h=\sqrt{300}=10\sqrt{3}$

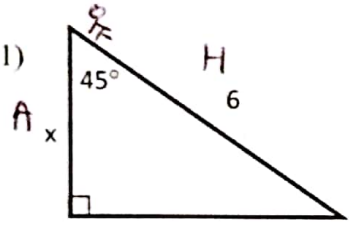
2. A side of a square is 4 cm long. What is the diagonal of the square in the **simplest radical form**? (Do you see a right triangle?)



$4^2+4^2=x^2$
$x^2=32$
$x=\sqrt{32}=4\sqrt{2}$

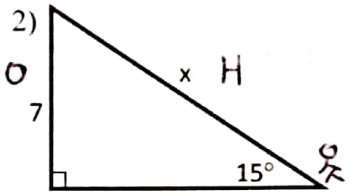
3. Solve for x. Round to the tenth.

1)



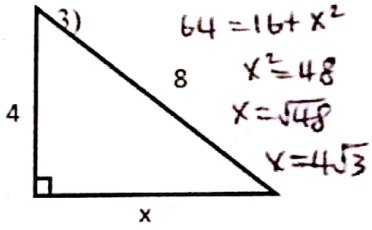
$\cos 45=\frac{x}{6}$ $x=6\cos 45$ $=4.2$

2)



$\sin 15=\frac{7}{x}$ $x=\frac{7}{\sin 15}=27.0$

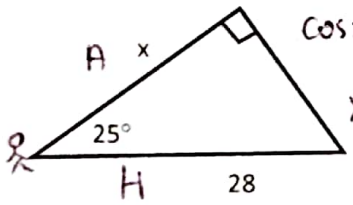
3)



$8^2=4^2+x^2$
$64=16+x^2$
$x^2=48$
$x=\sqrt{48}$
$x=4\sqrt{3}$

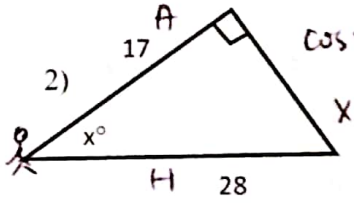
4. Solve for x.Round to the tenth.

1)



$\cos 25=\frac{x}{28}$ $x=28\cos 25$ $x=25.4$

2)



$\cos x=\frac{17}{28}$ $x=\cos^{-1}\frac{17}{28}=52.6$

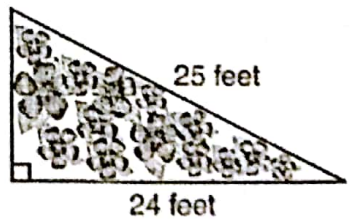
diagonal → $c^2=a^2+b^2$

5. Which window with the following dimensions is too small to allow a 35-inch piece of glass to fit through it?

A. 28 ×45 inches $c^2=28^2+45^2$ $c=53$ B. 16 ×33 inches $c^2=16^2+33^2$ $c=36.7$
C. 20 ×28 inches $c^2=20^2+28^2$ $c=34.4$ D. 40 ×42 inches $c^2=40^2+42^2$ $c=58$

diagonal is shorter than 35

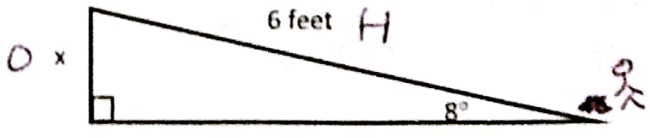
6. Stephen is planning a right triangular garden. He marked two sides that measure 24 feet and 25 feet. He wants to know the perimeter of the garden so that he can find out how many bricks he should buy at the store. Find the perimeter of the garden.



$n^2+24^2=25^2$
$n^2=625-576=49$
$n=7$

$7+24+25$
$=56$

7. The trainer adjusted a 6-**foot** long bench press so that the angle of elevation was 8°. How many **inches** did the trainer raise the bench press?

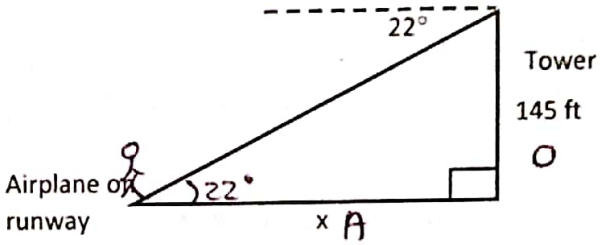


$\sin 8^\circ=\frac{x}{6}$
$x=6\sin 8^\circ=.8350$ ft
times 12 10.0 inches

8. From the top of a 145-foot high tower, an air traffic controller observes an airplane on the runway at an angle of depression of 22°. How far from the base of the tower is the airplane?

$\tan 22 = \frac{145}{x}$

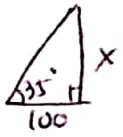
$x = \frac{145}{\tan 22} = 358.9$ ft



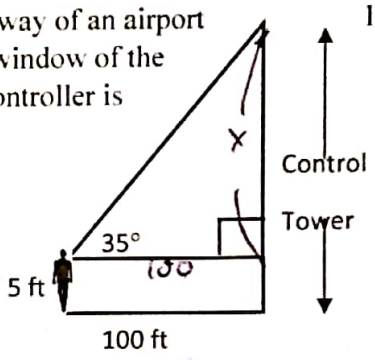
9. Chelsea whose eyes are 5 feet above the ground is standing on the runway of an airport 100 feet from the control tower. She observes an air traffic controller at the window of the control tower. The angle of elevation from the person to the air traffic controller is 35°. How tall is the control tower?

$\tan 35 = \frac{x}{100}$

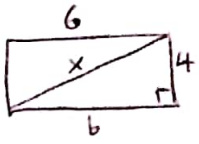
$x = 100 \tan 35 = 70$



tower height = eye height + x = 75 ft



10. Liz is building a rectangular gate. The dimensions of the gate are 6 feet high and 4 feet wide. She wants to fasten a thin brace diagonally at the corners to keep the gate sturdy. Approximately, how long is the brace?



$6^2 + 4^2 = x^2$

$36 + 16 = x^2$

$x^2 = 52$

$x = \sqrt{52}$

$4 \sqrt{13}$

$2\ 2$

$x = 2\sqrt{13}$

$a^2 + b^2 = c^2$

11. Rosemary is cutting 3 wooden sticks to build part of a kite frame. The part she is building must be a right triangle. Which choice below could be the lengths, in inches, of the sticks Rosemary cut? Choose all that apply.

A. 4, 5, 6 $4^2 + 5^2 \neq 6^2$

B. 4, 3, 5 $3^2 + 4^2 = 5^2$

C. 10, 15, 12 $10^2 + 12^2 \neq 15^2$

D. 12, 13, 5 $5^2 + 12^2 = 13^2$

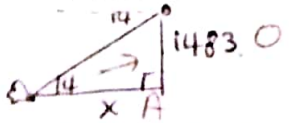
E. $\sqrt{7}, \sqrt{5}, 4$ $(\sqrt{7})^2 + (\sqrt{5})^2 = 12$, $4^2 = 16$

F. $\sqrt{3}, \sqrt{6}, 3$ $(\sqrt{3})^2 + (\sqrt{6})^2 = 9$, $3^2 = 9$

G. 9, 40, 41 $9^2 + 40^2 = 1681$, $41^2 = 1681$

H. $\sqrt{5}, 2\sqrt{2}, 13$ $(\sqrt{5})^2 + (2\sqrt{2})^2 = 13$, $13^2 = 169$

12. The angle of depression of an object on the ground is 14° from the top of the tallest building in the world, one of Petronas towers in Malaysia, which is 1,483 feet high. What is the distance from the object to the base of the tower to the nearest foot?

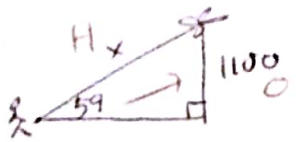


$\tan 14 = \frac{1483}{x}$

$x = \frac{1483}{\tan 14} = 5947.988\ldots$

5948 ft

13. You are at the air show in Virginia Beach. You are looking up at a British Harrier Jet at an angle of elevation of 59°. If the plane is hovering 1100 ft above the water, how far are you from the jet (direct distance)? Round to the nearest foot.

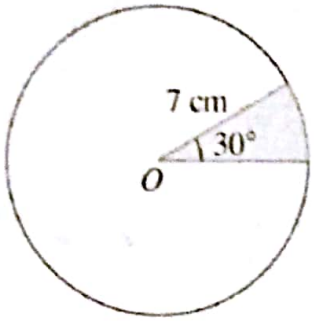


$\sin 59 = \frac{1100}{x}$

$x = \frac{1100}{\sin 59} = 1283.2967\ldots$

1283 ft

Finding Arc Length & Area of Sector



$$Arc\ Length = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{30^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 7cm$$

$$= 3.667\ cm$$

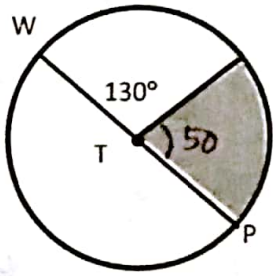
$$Area\ of\ Sector = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 7^2$$

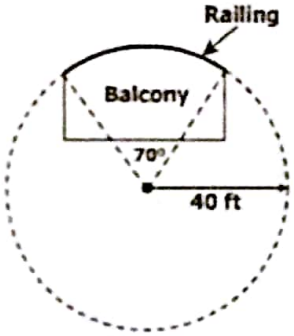
$$= 12.83\ cm^2$$

diameter radius = 18

1. Given circle T with WP = 36 cm. Calculate the exact **area** of the shaded sector.



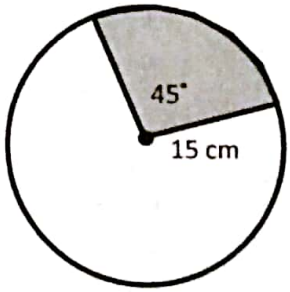
$\pi r^2 \frac{\theta}{360} = \pi(18)^2\left(\frac{50}{360}\right) =$ 141.37



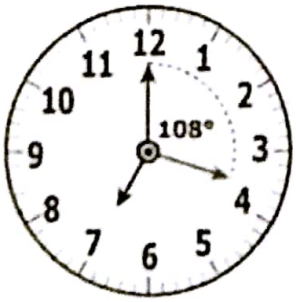
2. Find the length of the balcony. arc

$2\pi r\left(\frac{\theta}{360}\right) = 2\pi(40)\frac{70}{360} =$ 48.87

3. Calculate the area of the shaded sector, to the nearest tenth.



$\pi r^2\left(\frac{\theta}{360}\right) = \pi(15^2)\frac{45}{360} =$ 88.36



4. The minute hand on a clock is 10 centimeters long and travels through an arc of 108° every 18 minutes. Find the measure of the length of the arc the minute hand travels through during this 18-minute period.

$2\pi r\left(\frac{\theta}{360}\right) = 2\pi(10)\left(\frac{108}{360}\right) =$ 18.85